高阶导数

1. 二阶导数：对一阶导数再求导. 记为 $f''(x)$

记号：$f''(x) = \frac{d^2f}{dx^2} = \frac{d^2y}{dx^2} = y''$

$f'''(x) = \frac{d^3f}{dx^3} = \frac{d^3y}{dx^3} = y'''$

$f^{(n)}(x) = \frac{d^nf}{dx^n} = \frac{d^ny}{dx^n} = y^{(n)}$

例1. 求 $y = x\sin x$ 的二阶导数

解：$y' = \sin x + x\cos x$

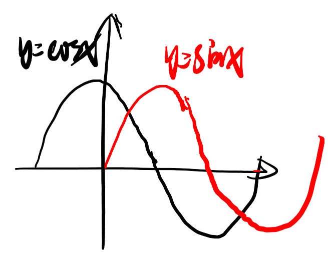
$y'' = \cos x + \cos x - x\sin x = 2\cos x - x\sin x$

例2. 求 $y = \sin(e^x)$ 的二阶导数

解：$y' = \cos(e^x)\cdot e^x$，$y'' = -\sin(e^x)\cdot e^x\cdot e^x + \cos(e^x)e^x$

$= -e^{2x}\sin(e^x) + e^x\cos(e^x)$

例3. 求 $y=\sin x$ 的 n 阶导数



解：$y' = \cos x = \sin(x+\frac{\pi}{2})$

$y'' = \cos(x+\frac{\pi}{2}) = \sin(x+\frac{\pi}{2}+\frac{\pi}{2}) = \sin(x+\pi)$

$y''' = \cos(x+\pi) = \sin(x+\pi+\frac{\pi}{2}) = \sin(x+\frac{3}{2}\pi)$

$\vdots$

$y^{(n)} = \sin(x+\frac{n}{2}\pi)$

2. 隐函数的高阶导数.（一阶用隐函数求导，高阶用复合函数求导）

例4. $xy=1$ 求 $\frac{d^2y}{dx^2}$

解：$y + xy' = 0 \Rightarrow y' = -\frac{y}{x} \Rightarrow \boxed{\frac{dy}{dx} = -\frac{y}{x}}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{\frac{dy}{dx}\cdot x - \frac{dy}{dx}\cdot 1}{x^2} = -\frac{-\frac{y}{x}\cdot x - (-\frac{y}{x})}{x^2}$

$$= \frac{y-\frac{y}{x}}{x^2} = \frac{xy-y}{x^3} = \frac{1-y}{x^3}$$

3、参数的高阶导数（一阶用参数方程求导法，高阶用复合函数求导法）

例5、$x=\sec t$, $y=\tan t$，求 $\frac{d^2y}{dx^2}$

解：$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\sec^2 t}{\sec t\tan t} = \frac{\sec t}{\tan t} = \frac{1}{\cos t}\cdot\frac{\cos t}{\sin t} = \frac{1}{\sin t}$

$= \csc t$.

$$\boxed{\frac{dt}{dx}\cdot\frac{d}{dt} = \frac{d}{dx}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{1}{\frac{dx}{dt}}$$

$$= (\csc t)'\cdot\frac{1}{x'(t)} = -\csc t\cot t\cdot\frac{1}{\sec t\tan t}$$

$$= -\frac{1}{\sin t}\cdot\frac{\cos t}{\sin t}\cdot\cos t\cdot\frac{\cos t}{\sin t}$$

$$= -\frac{\cos^3 t}{\sin^3 t} = -\cot^3 t$$

参数方程的二阶导数：$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\Big/\frac{dx}{dt}$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right)\Big/x'(t)$$