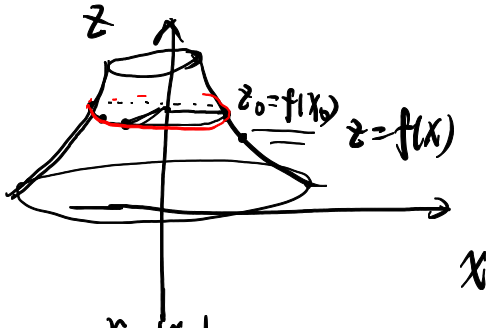
旋转面、柱面与其它二次曲面

1. 旋转面：① $z=f(\pm\sqrt{x^2+y^2})$ ①

$F(z,\pm\sqrt{x^2+y^2})=0$ ②



半径 $|x_0|$，圆的方程：$y^2+z^2=x_0^2 \Rightarrow x_0=\pm\sqrt{y^2+z^2}$

$z=f(\pm\sqrt{x^2+y^2})$

$z=f(x)$ 或者 $z=f(y)$：旋转面的母线

z-轴：旋转轴

② $y=g(\pm\sqrt{x^2+z^2})$

$G(y,\pm\sqrt{x^2+z^2})=0$

母线：$y=g(x)$ 旋转轴：y-轴

$G(y,z)=0$

$G(y,x)=0$

③ $x=h(\pm\sqrt{y^2+z^2})$

$H(x,\pm\sqrt{y^2+z^2})=0$

母线：$x=h(y)$ 旋转轴：x-轴

$H(x,0)=0$

例1. $z=x^2+y^2$ $z=x^2$ 绕z轴旋转

$z=(\sqrt{x^2+y^2})^2$

例2. $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{9}=1$，$\frac{x^2}{4}+\frac{(\sqrt{y^2+z^2})^2}{9}=1$

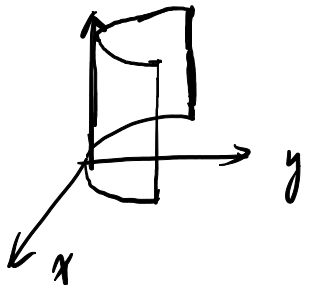
$\frac{x^2}{4}+\frac{y^2}{9}=1$ 绕x轴旋转

母线。

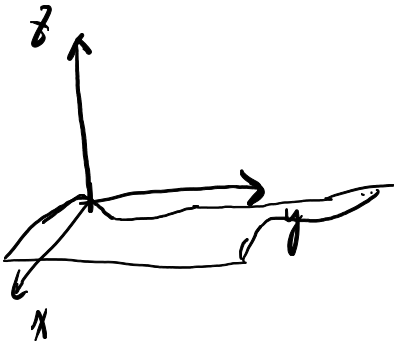
2. 柱面：方程缺少一个变量，少的变量为平行轴

$y=x^2$：准线

母线：平行于z轴的直线叫母线



$x=z^3$



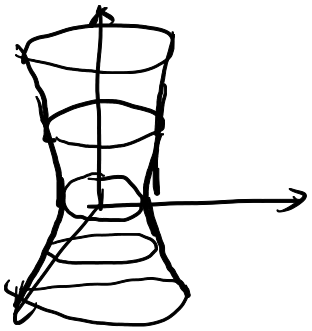
3. 其它二次曲面（水平线，截痕法，含z=常数，y=常数，x=常数）

① 椭圆锥面：$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z^2$

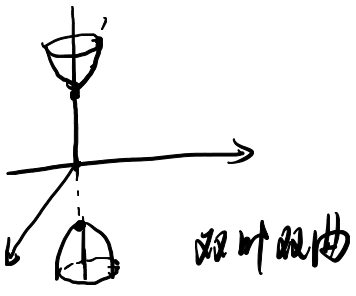
② 椭球面：$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

③ 单叶双曲面：$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

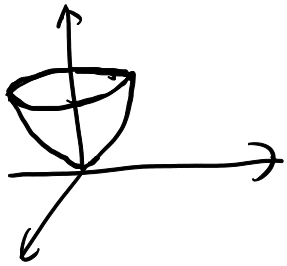
$z=0$



④ 双叶双曲面：$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$



⑤ 椭圆抛物面：$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$



⑥ 双曲抛物面

$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$