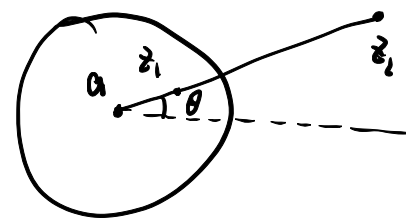


几个特殊的分式线性函数

1. 对称点的表达式: z_1, z_2 关于以 a 为圆心的圆对称



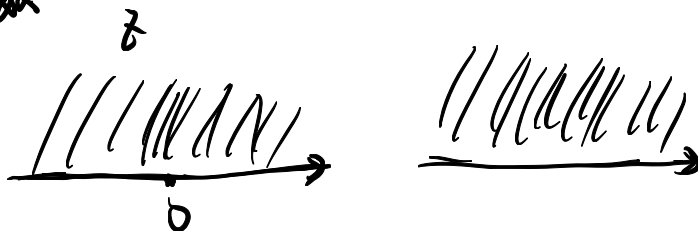
$$z_1 - a = r e^{i\theta}$$

$$z_2 - a = \frac{R^2}{r} e^{i\theta} = \frac{R^2}{r e^{-i\theta}} = \frac{R^2}{z_1 - a}$$

$\Rightarrow z_2 - a = \frac{R^2}{z_1 - a}$ 单位圆 (圆心在原点) $z_2 = \frac{1}{z_1}$

2. 将上半平面变成上半平面的分式线性函数

- ① 实数轴 \rightarrow 实数轴
- ② 方向一致



$$w = \frac{az+b}{cz+d}$$

z 为实数 $\Rightarrow w$ 为实数 $\Rightarrow a, b, c, d$ 都是实数

实数轴上, z 增加, w 增加

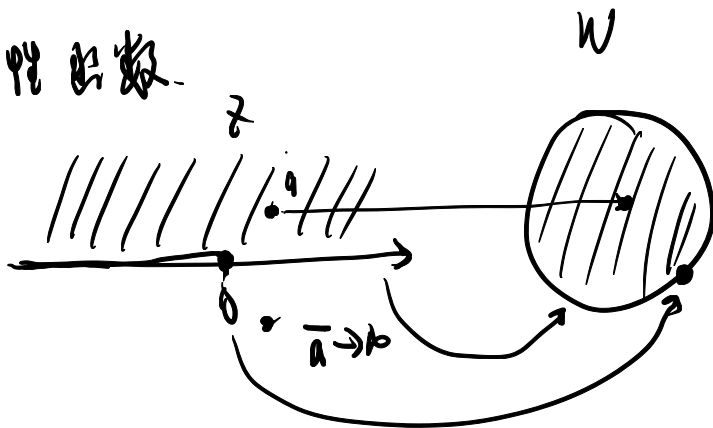
$$w' = \frac{a(cz+d) - c(az+b)}{(cz+d)^2} = \frac{ad-bc}{(cz+d)^2} > 0 \Rightarrow ad-bc > 0$$

$\Rightarrow w = \frac{az+b}{cz+d}, a, b, c, d \in \mathbb{R}, ad-bc > 0$

将上半平面变成上半平面 (下半平面变成下半平面)

3. 将上半平面变为单位圆的分式线性函数

$a \rightarrow 0$
 $\bar{a} \rightarrow \infty$



$$\Rightarrow w = k \frac{z-a}{z-\bar{a}}$$

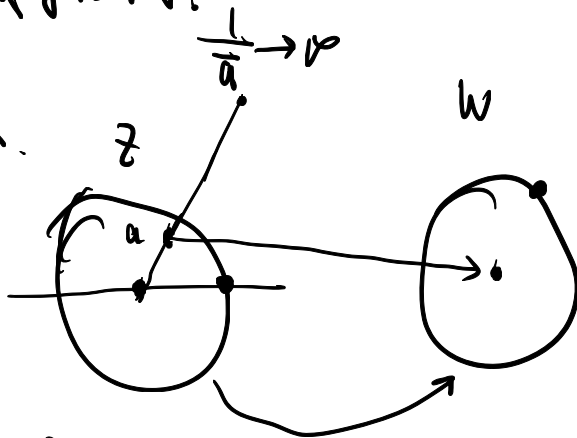
$\Rightarrow z=0$ 时, $|w|=1 = \left| k \frac{a}{\bar{a}} \right| = |k| \left| \frac{a}{\bar{a}} \right| = |k| \Rightarrow |k|=1 \quad k = e^{i\theta}$

$$\Rightarrow w = e^{i\theta} \frac{z-a}{z-\bar{a}} \quad \text{将上半平面变成单位圆.}$$

4. 将单位圆变成单位圆的分式线性函数.

$$a \rightarrow 0$$

$$\frac{1}{a} \rightarrow \infty$$



$$w = k \frac{z-a}{z-\frac{1}{\bar{a}}} = \bar{a}k \frac{z-a}{\bar{a}z-1} = -\bar{a}k \frac{z-a}{1-\bar{a}z}$$

$$|w| = 1 = |k\bar{a} \cdot \frac{1-a}{\bar{a}-1}| = |k\bar{a}| \left| \frac{1-a}{\bar{a}-1} \right| = |k\bar{a}| = 1$$

$$\Rightarrow -k\bar{a} = e^{i\theta}$$

$$\Rightarrow w = e^{i\theta} \frac{z-a}{1-\bar{a}z} \quad \text{将单位圆变成单位圆 (|a| < 1.)}$$