复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = \tan(\ln(x))$

b) $f(x) = \ln(\csc(x))$

c) $f(x) = \csc(e^x)$

d) $f(x) = \tan(\cot(x))$

e) $f(x) = e^{\cot(x)}$

f) $f(x) = \cot(\cot(x))$

g) $f(x) = \operatorname{arcsec}^2(x)$

h) $f(x) = \ln(\arccos(x))$

i) $f(x) = \operatorname{arcsec}(\arctan(x))$

j) $f(x) = \ln(\ln(x))$

k) $f(x) = \ln(x)^2$

l) $f(x) = \arcsin(\text{arcsec}(x))$

m) $f(x) = \ln(\arcsin(x))$

n) $f(x) = \arcsin(\arctan(x))$

o) $f(x) = \cos\left(x^3\right)$

p) $f(x) = \arctan(\cos(x))$

q) $f(x) = \dfrac{1}{x}$

r) $f(x) = \text{arccot}(e^x)$

2. 求下列函数的导数：

a) $f(x) = \tan(\sec(\sin(x)))$

b) $f(x) = \csc(x)$

c) $f(x) = \ln(\sec(\ln(x)))$

d) $f(x) = e^{\cot(e^x)}$

e) $f(x) = \operatorname{arcsin}^2(e^x)$

f) $f(x) = \operatorname{arcsec}^4(x)$

g) $f(x) = \arctan(\ln(\arcsin(x)))$

h) $f(x) = x^2$

i) $f(x) = \operatorname{arccsc}(\cos(\sin(x)))$

j) $f(x) = \cot\left(e^{\csc(x)}\right)$

k) $f(x) = \operatorname{arccot}\left(\tan^3(x)\right)$

l) $f(x) = \operatorname{arccot}(\operatorname{arcsec}(\cot(x)))$

3. 求下列函数的导数：

a) $f(x) = \sqrt{\sqrt{x} + \ln(x)}$

b) $f(x) = \sqrt{\tan(x) + \arctan(x)}$

c) $f(x) = \cot(\cos(x) + \operatorname{arccot}(x))$

d) $f(x) = \tan(2\tan(x))$

e) $f(x) = \sqrt{\cot(x) + \operatorname{arccot}(x)}$

f) $f(x) = \ln(e^x + \sin(x))$

g) $f(x) = \arccos(\ln(x)\cot(x))$

h) $f(x) = \arcsin(e^x \ln(x))$

i) $f(x) = \arctan\left(\sqrt{x}\sin(x)\right)$

j) $f(x) = e^{\sqrt{x}\arccos(x)}$

k) $f(x) = \ln\left(\dfrac{\cos(x)}{\arccos(x)}\right)$

l) $f(x) = \tan\left(e^{-x}\tan(x)\right)$

m) $f(x) = \sqrt{\dfrac{\ln(x)}{\sin(x)}}$

n) $f(x) = \arcsin\left(\dfrac{\ln(x)}{\cot(x)}\right)$

4. 求下列函数的导数：

a) $f(x) = e^{\sin(x)+\operatorname{arccot}(\cot(x))}$

b) $f(x) = \arcsin(e^x + \ln(\arctan(x)))$

c) $f(x) = \tan(\cos(\cot(x)) + \tan(x))$

d) $f(x) = \cos(\tan(\ln(x)) + \cot(x))$

e) $f(x) = \arcsin(\ln(x) + \operatorname{arccot}(\arccos(x)))$

f) $f(x) = \arctan\left(e^{\tan(x)} + \cot(x)\right)$

g) $f(x) = \sin(\arcsin(x)\arctan(e^x))$

h) $f(x) = \ln\left(\ln(x)\sqrt{\operatorname{arccot}(x)}\right)$

i) $f(x) = \ln\left(\sqrt{1-x^2}\arcsin(x)\right)$

j) $f(x) = \cot(\tan(x)\tan(\tan(x)))$

答案

1.

a) $f'(x)=\frac{\tan^2(\ln(x))+1}{x}$

b) $f'(x)=-\cot(x)$

c) $f'(x)=-e^x\cot(e^x)\csc(e^x)$

d) $f'(x)=\left(\tan^2(\cot(x))+1\right)\left(-\cot^2(x)-1\right)$

e) $f'(x)=\left(-\cot^2(x)-1\right)e^{\cot(x)}$

f) $f'(x)=\left(-\cot^2(x)-1\right)\left(-\cot^2(\cot(x))-1\right)$

g) $f'(x)=\frac{2\,\text{arcsec}(x)}{x^2\sqrt{1-\frac{1}{x^2}}}$

h) $f'(x)=-\frac{1}{\sqrt{1-x^2}\arccos(x)}$

i) $f'(x)=\frac{1}{\sqrt{1-\frac{1}{\arctan^2(x)}}\,(x^2+1)\arctan^2(x)}$

j) $f'(x)=\frac{1}{x\ln(x)}$

k) $f'(x)=\frac{2\ln(x)}{x}$

l) $f'(x)=\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\text{arcsec}^2(x)}}$

m) $f'(x)=\frac{1}{\sqrt{1-x^2}\arcsin(x)}$

n) $f'(x)=\frac{1}{\sqrt{1-\arctan^2(x)}\,(x^2+1)}$

o) $f'(x)=-3x^2\sin\left(x^3\right)$

p) $f'(x)=-\frac{\sin(x)}{\cos^2(x)+1}$

q) $f'(x)=-\frac{1}{x^2}$

r) $f'(x)=-\frac{e^x}{e^{2x}+1}$

2.

a) $f'(x)=\left(\tan^2(\sec(\sin(x)))+1\right)\cos(x)\tan(\sin(x))\sec(\sin(x))$

b) $f'(x)=-\cot(x)\csc(x)$

c) $f'(x)=\frac{\tan(\ln(x))}{x}$

d) $f'(x)=\left(-\cot^2(e^x)-1\right)e^x e^{\cot(e^x)}$

e) $f'(x)=\frac{2e^x\arcsin(e^x)}{\sqrt{1-e^{2x}}}$

f) $f'(x)=\frac{4\,\text{arcsec}^3(x)}{x^2\sqrt{1-\frac{1}{x^2}}}$

g) $f'(x)=\frac{1}{\sqrt{1-x^2}\left(\ln(\arcsin(x))^2+1\right)\arcsin(x)}$

h) $f'(x)=2x$

i) $f'(x) = \frac{\sin(\sin(x))\cos(x)}{\sqrt{1 - \frac{1}{\cos^2(\sin(x))}}\cos^2(\sin(x))}$

j) $f'(x) = -\left(-\cot^2\left(e^{\csc(x)}\right) - 1\right)e^{\csc(x)}\cot(x)\csc(x)$

k) $f'(x) = -\frac{(3\tan^2(x) + 3)\tan^2(x)}{\tan^6(x) + 1}$

l) $f'(x) = -\frac{-\cot^2(x) - 1}{\sqrt{1 - \frac{1}{\cot^2(x)}}\left(\operatorname{arcsec}^2(\cot(x)) + 1\right)\cot^2(x)}$

3.

a) $f'(x) = \frac{\frac{1}{2x} + \frac{1}{4\sqrt{x}}}{\sqrt{\sqrt{x} + \ln(x)}}$

b) $f'(x) = \frac{\frac{\tan^2(x)}{2} + \frac{1}{2} + \frac{1}{2(x^2+1)}}{\sqrt{\tan(x) + \arctan(x)}}$

c) $f'(x) = \left(-\sin(x) - \frac{1}{x^2 + 1}\right)\left(-\cot^2(\cos(x) + \operatorname{arccot}(x)) - 1\right)$

d) $f'(x) = \left(2\tan^2(x) + 2\right)\left(\tan^2(2\tan(x)) + 1\right)$

e) $f'(x) = \frac{-\frac{\cot^2(x)}{2} - \frac{1}{2} - \frac{1}{2(x^2+1)}}{\sqrt{\cot(x) + \operatorname{arccot}(x)}}$

f) $f'(x) = \frac{e^x + \cos(x)}{e^x + \sin(x)}$

g) $f'(x) = -\frac{(-\cot^2(x) - 1)\ln(x) + \frac{\cot(x)}{x}}{\sqrt{-\ln(x)^2\cot^2(x) + 1}}$

h) $f'(x) = \frac{e^x\ln(x) + \frac{e^x}{x}}{\sqrt{-e^{2x}\ln(x)^2 + 1}}$

i) $f'(x) = \frac{\sqrt{x}\cos(x) + \frac{\sin(x)}{2\sqrt{x}}}{x\sin^2(x) + 1}$

j) $f'(x) = \left(-\frac{\sqrt{x}}{\sqrt{1 - x^2}} + \frac{\arccos(x)}{2\sqrt{x}}\right)e^{\sqrt{x}\arccos(x)}$

k) $f'(x) = \frac{\left(-\frac{\sin(x)}{\arccos(x)} + \frac{\cos(x)}{\sqrt{1-x^2}\arccos^2(x)}\right)\arccos(x)}{\cos(x)}$

l) $f'(x) = \left(\left(\tan^2(x) + 1\right)e^{-x} - e^{-x}\tan(x)\right)\left(\tan^2\left(e^{-x}\tan(x)\right) + 1\right)$

m) $f'(x) = \dfrac{\sqrt{\frac{\ln(x)}{\sin(x)}}\left(-\frac{\ln(x)\cos(x)}{2\sin^2(x)} + \frac{1}{2x\sin(x)}\right)\sin(x)}{\ln(x)}$

n) $f'(x) = \dfrac{\frac{\left(\cot^2(x)+1\right)\ln(x)}{\cot^2(x)} + \frac{1}{x\cot(x)}}{\sqrt{-\frac{\ln(x)^2}{\cot^2(x)} + 1}}$

4.

a) $f'(x) = \left(-\dfrac{-\cot^2(x) - 1}{\cot^2(x) + 1} + \cos(x)\right) e^{\sin(x) + \operatorname{arccot}(\cot(x))}$

b) $f'(x) = \dfrac{e^x + \frac{1}{(x^2+1)\arctan(x)}}{\sqrt{1 - (e^x + \ln(\arctan(x)))^2}}$

c) $f'(x) = \left(\tan^2(\cos(\cot(x)) + \tan(x)) + 1\right)\left(-\left(-\cot^2(x) - 1\right)\sin(\cot(x)) + \tan^2(x) + 1\right)$

d) $f'(x) = -\left(-\cot^2(x) - 1 + \dfrac{\tan^2(\ln(x)) + 1}{x}\right)\sin(\tan(\ln(x)) + \cot(x))$

e) $f'(x) = \dfrac{\frac{1}{\sqrt{1-x^2}(\arccos^2(x)+1)} + \frac{1}{x}}{\sqrt{1 - (\ln(x) + \operatorname{arccot}(\arccos(x)))^2}}$

f) $f'(x) = \dfrac{(\tan^2(x) + 1)\, e^{\tan(x)} - \cot^2(x) - 1}{\left(e^{\tan(x)} + \cot(x)\right)^2 + 1}$

g) $f'(x) = \left(\dfrac{e^x \arcsin(x)}{e^{2x} + 1} + \dfrac{\arctan(e^x)}{\sqrt{1 - x^2}}\right)\cos(\arcsin(x)\arctan(e^x))$

h) $f'(x) = \dfrac{-\frac{\ln(x)}{2(x^2+1)\sqrt{\operatorname{arccot}(x)}} + \frac{\sqrt{\operatorname{arccot}(x)}}{x}}{\ln(x)\sqrt{\operatorname{arccot}(x)}}$

i) $f'(x) = \dfrac{-\frac{x\arcsin(x)}{\sqrt{1-x^2}} + 1}{\sqrt{1 - x^2}\arcsin(x)}$

j) $f'(x) = \left(\left(\tan^2(x) + 1\right)\left(\tan^2(\tan(x)) + 1\right)\tan(x) + \left(\tan^2(x) + 1\right)\tan(\tan(x))\right)\left(-\cot^2(\tan(x)\tan\right.$