复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = \sqrt{\ln(x)}$

b) $f(x) = e^{\cot(x)}$

c) $f(x) = \cot(\sec(x))$

d) $f(x) = \sqrt{\tan(x)}$

e) $f(x) = \cos(\csc(x))$

f) $f(x) = \csc(\sin(x))$

g) $f(x) = \operatorname{arccsc}(\operatorname{arcsec}(x))$

h) $f(x) = e^{\arctan(x)}$

i) $f(x) = \arccos(\ln(x))$

j) $f(x) = \operatorname{arccsc}(\arctan(x))$

k) $f(x) = \ln(e^x)$

l) $f(x) = \arcsin(\ln(x))$

m) $f(x) = \operatorname{arcsec}(\tan(x))$

n) $f(x) = \tan(e^x)$

o) $f(x) = \arccos(\csc(x))$

p) $f(x) = \arcsin\left(x^3\right)$

q) $f(x) = \operatorname{arccsc}(\sin(x))$

r) $f(x) = \sin(\cos(x))$

2. 求下列函数的导数：

a) $f(x) = e^{\tan(\ln(x))}$

b) $f(x) = \sin(\csc(\cos(x)))$

c) $f(x) = \sin\left(\sqrt{\cos(x)}\right)$

d) $f(x) = e^{\sqrt{\ln(x)}}$

e) $f(x) = \arctan(\operatorname{arcsec}(\operatorname{arcsec}(x)))$

f) $f(x) = e^{\operatorname{arcsec}(\arccos(x))}$

g) $f(x) = \operatorname{arcsec}(\arcsin(\operatorname{arccot}(x)))$

h) $f(x) = \ln(\arccos(\operatorname{arccsc}(x)))$

i) $f(x) = \operatorname{arcsec}(\operatorname{arccot}(\operatorname{arccsc}(x)))$

j) $f(x) = \cos\left(\frac{1}{x}\right)$

k) $f(x) = \csc\left(\frac{1}{\sqrt{1-x^2}}\right)$

l) $f(x) = \frac{1}{\sqrt{1 - \frac{1}{\operatorname{arccot}^2(x)}}\operatorname{arccot}(x)}$

3. 求下列函数的导数：

a) $f(x) = \sqrt{\ln(x) + \arctan(x)}$

b) $f(x) = \sin(2e^x)$

c) $f(x) = \cos(\cos(x) + \arcsin(x))$

d) $f(x) = \cot(\cos(x) + \arcsin(x))$

e) $f(x) = \arctan\left(\sqrt{x} + \sin(x)\right)$

f) $f(x) = e^{\cot(x) + \arctan(x)}$

g) $f(x) = \sqrt{\sin(x)\cos(x)}$

h) $f(x) = \sin(\cos(x)\operatorname{arccot}(x))$

i) $f(x) = \arctan(e^{x}\sin(x))$

j) $f(x) = \ln(\cot(x)\operatorname{arccot}(x))$

k) $f(x) = \arcsin\left(e^{-x}\arcsin(x)\right)$

l) $f(x) = \sin\left(\frac{\tan(x)}{\sin(x)}\right)$

m) $f(x) = \arctan\left(\frac{\tan(x)}{\sqrt{x}}\right)$

n) $f(x) = \sin\left(\frac{e^{x}}{\operatorname{arccot}(x)}\right)$

4. 求下列函数的导数：

a) $f(x) = \sin\left(\sqrt{\tan(x)} + \arcsin(x)\right)$

b) $f(x) = \arccos\left(\tan(x) + \sqrt{\operatorname{arccot}(x)}\right)$

c) $f(x) = e^{e^{\arctan(x)} + \cos(x)}$

d) $f(x) = \tan\left(\sin(x) + \arctan\left(\sqrt{x}\right)\right)$

e) $f(x) = \arccos(\arcsin(x) + \arcsin(\cos(x)))$

f) $f(x) = \ln(\ln(\cot(x)) + \cos(x))$

g) $f(x) = \sqrt{\arcsin(x)\sqrt{\arctan(x)}}$

h) $f(x) = \tan\left(\frac{\operatorname{arccot}(x)}{x}\right)$

i) $f(x) = \cot\left(\frac{e^x}{x\sqrt{1 + \frac{1}{x^2}}}\right)$

j) $f(x) = \arctan\left(\cos(x)\arctan\left(\sqrt{x}\right)\right)$

答案

1.

a) $f'(x) = \frac{1}{2x\sqrt{\ln(x)}}$

b) $f'(x) = \left(-\cot^2(x) - 1\right) e^{\cot(x)}$

c) $f'(x) = \left(-\cot^2(\sec(x)) - 1\right)\tan(x)\sec(x)$

d) $f'(x) = \frac{\frac{\tan^2(x)}{2} + \frac{1}{2}}{\sqrt{\tan(x)}}$

e) $f'(x) = \sin(\csc(x))\cot(x)\csc(x)$

f) $f'(x) = -\cos(x)\cot(\sin(x))\csc(\sin(x))$

g) $f'(x) = -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{\operatorname{arcsec}^2(x)}}\operatorname{arcsec}^2(x)}$

h) $f'(x) = \frac{e^{\arctan(x)}}{x^2+1}$

i) $f'(x) = -\frac{1}{x\sqrt{1-\ln(x)^2}}$

j) $f'(x) = -\frac{1}{\sqrt{1-\frac{1}{\arctan^2(x)}}\,(x^2+1)\arctan^2(x)}$

k) $f'(x) = 1$

l) $f'(x) = \frac{1}{x\sqrt{1-\ln(x)^2}}$

m) $f'(x) = \frac{\tan^2(x)+1}{\sqrt{1-\frac{1}{\tan^2(x)}}\tan^2(x)}$

n) $f'(x) = \left(\tan^2(e^x)+1\right)e^x$

o) $f'(x) = \frac{\cot(x)\csc(x)}{\sqrt{1-\csc^2(x)}}$

p) $f'(x) = \frac{3x^2}{\sqrt{1-x^6}}$

q) $f'(x) = -\frac{\cos(x)}{\sqrt{1-\frac{1}{\sin^2(x)}}\sin^2(x)}$

r) $f'(x) = -\sin(x)\cos(\cos(x))$

2.

a) $f'(x) = \frac{(\tan^2(\ln(x))+1)\,e^{\tan(\ln(x))}}{x}$

b) $f'(x) = \sin(x)\cos(\csc(\cos(x)))\cot(\cos(x))\csc(\cos(x))$

c) $f'(x) = -\frac{\sin(x)\cos\left(\sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)}}$

d) $f'(x) = \frac{e^{\sqrt{\ln(x)}}}{2x\sqrt{\ln(x)}}$

e) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{\operatorname{arcsec}^2(x)}}\,(\operatorname{arcsec}^2(\operatorname{arcsec}(x))+1)\operatorname{arcsec}^2(x)}$

f) $f'(x) = -\frac{e^{\operatorname{arcsec}(\arccos(x))}}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arccos^2(x)}}\arccos^2(x)}$

g) $f'(x) = -\frac{1}{\sqrt{1-\operatorname{arccot}^2(x)}\sqrt{1-\frac{1}{\arcsin^2(\operatorname{arccot}(x))}}(x^2+1)\arcsin^2(\operatorname{arccot}(x))}$

h) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arccsc}^2(x)}\arccos(\operatorname{arccsc}(x))}$

i) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{\operatorname{arccot}^2(\operatorname{arccsc}(x))}}(\operatorname{arccsc}^2(x)+1)\operatorname{arccot}^2(\operatorname{arccsc}(x))}$

j) $f'(x) = \frac{\sin\left(\frac{1}{x}\right)}{x^2}$

k) $f'(x) = -\frac{x\cot\left(\frac{1}{\sqrt{1-x^2}}\right)\csc\left(\frac{1}{\sqrt{1-x^2}}\right)}{(1-x^2)^{\frac{3}{2}}}$

l) $f'(x) = \frac{1}{\sqrt{1-\frac{1}{\operatorname{arccot}^2(x)}}(x^2+1)\operatorname{arccot}^2(x)} + \frac{1}{\left(1-\frac{1}{\operatorname{arccot}^2(x)}\right)^{\frac{3}{2}}(x^2+1)\operatorname{arccot}^4(x)}$

3.

a) $f'(x) = \frac{\frac{1}{2(x^2+1)}+\frac{1}{2x}}{\sqrt{\ln(x)+\arctan(x)}}$

b) $f'(x) = 2e^x\cos(2e^x)$

c) $f'(x) = -\left(-\sin(x)+\frac{1}{\sqrt{1-x^2}}\right)\sin(\cos(x)+\arcsin(x))$

d) $f'(x) = \left(-\sin(x)+\frac{1}{\sqrt{1-x^2}}\right)\left(-\cot^2(\cos(x)+\arcsin(x))-1\right)$

e) $f'(x) = \frac{\cos(x)+\frac{1}{2\sqrt{x}}}{(\sqrt{x}+\sin(x))^2+1}$

f) $f'(x) = \left(-\cot^2(x)-1+\frac{1}{x^2+1}\right)e^{\cot(x)+\arctan(x)}$

g) $f'(x) = \frac{\sqrt{\sin(x)\cos(x)}\left(-\frac{\sin^2(x)}{2}+\frac{\cos^2(x)}{2}\right)}{\sin(x)\cos(x)}$

h) $f'(x) = \left(-\sin(x)\operatorname{arccot}(x)-\frac{\cos(x)}{x^2+1}\right)\cos(\cos(x)\operatorname{arccot}(x))$

i) $f'(x) = \frac{e^x\sin(x)+e^x\cos(x)}{e^{2x}\sin^2(x)+1}$

j) $f'(x) = \frac{(-\cot^2(x)-1)\operatorname{arccot}(x)-\frac{\cot(x)}{x^2+1}}{\cot(x)\operatorname{arccot}(x)}$

k) $f'(x) = \dfrac{-e^{-x}\arcsin(x) + \frac{e^{-x}}{\sqrt{1-x^2}}}{\sqrt{1-e^{-2x}\arcsin^2(x)}}$

l) $f'(x) = \left(\dfrac{\tan^2(x)+1}{\sin(x)} - \dfrac{\cos(x)\tan(x)}{\sin^2(x)}\right)\cos\left(\dfrac{\tan(x)}{\sin(x)}\right)$

m) $f'(x) = \dfrac{\frac{\tan^2(x)+1}{\sqrt{x}} - \frac{\tan(x)}{2x^{\frac{3}{2}}}}{1+\frac{\tan^2(x)}{x}}$

n) $f'(x) = \left(\dfrac{e^x}{\operatorname{arccot}(x)} + \dfrac{e^x}{(x^2+1)\operatorname{arccot}^2(x)}\right)\cos\left(\dfrac{e^x}{\operatorname{arccot}(x)}\right)$

4.

a) $f'(x) = \left(\dfrac{\frac{\tan^2(x)}{2}+\frac{1}{2}}{\sqrt{\tan(x)}} + \dfrac{1}{\sqrt{1-x^2}}\right)\cos\left(\sqrt{\tan(x)} + \arcsin(x)\right)$

b) $f'(x) = -\dfrac{\tan^2(x)+1-\frac{1}{2(x^2+1)\sqrt{\operatorname{arccot}(x)}}}{\sqrt{1-\left(\tan(x)+\sqrt{\operatorname{arccot}(x)}\right)^2}}$

c) $f'(x) = \left(-\sin(x) + \dfrac{e^{\arctan(x)}}{x^2+1}\right)e^{e^{\arctan(x)}+\cos(x)}$

d) $f'(x) = \left(\cos(x) + \dfrac{1}{2\sqrt{x}(x+1)}\right)\left(\tan^2\left(\sin(x)+\arctan\left(\sqrt{x}\right)\right)+1\right)$

e) $f'(x) = -\dfrac{-\frac{\sin(x)}{\sqrt{1-\cos^2(x)}} + \frac{1}{\sqrt{1-x^2}}}{\sqrt{1-(\arcsin(x)+\arcsin(\cos(x)))^2}}$

f) $f'(x) = \dfrac{\frac{-\cot^2(x)-1}{\cot(x)} - \sin(x)}{\ln(\cot(x)) + \cos(x)}$

g) $f'(x) = \dfrac{\sqrt{\arcsin(x)\sqrt{\arctan(x)}}\left(\frac{\arcsin(x)}{4(x^2+1)\sqrt{\arctan(x)}} + \frac{\sqrt{\arctan(x)}}{2\sqrt{1-x^2}}\right)}{\arcsin(x)\sqrt{\arctan(x)}}$

h) $f'(x) = \left(-\dfrac{1}{x(x^2+1)} - \dfrac{\operatorname{arccot}(x)}{x^2}\right)\left(\tan^2\left(\dfrac{\operatorname{arccot}(x)}{x}\right)+1\right)$

i) $f'(x) = \left(-\cot^2\left(\dfrac{e^x}{x\sqrt{1+\frac{1}{x^2}}}\right)-1\right)\left(\dfrac{e^x}{x\sqrt{1+\frac{1}{x^2}}} - \dfrac{e^x}{x^2\sqrt{1+\frac{1}{x^2}}} + \dfrac{e^x}{x^4\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}}\right)$

j) $f'(x) = \dfrac{-\sin(x)\arctan(\sqrt{x}) + \frac{\cos(x)}{2\sqrt{x}(x+1)}}{\cos^2(x)\arctan^2(\sqrt{x})+1}$