# 复合函数的求导法则

## 练习

1. 求下列函数的导数：

a) $f(x) = \sec(\csc(x))$

b) $f(x) = e^{e^x}$

c) $f(x) = \cot(e^x)$

d) $f(x) = \sin(\cot(x))$

e) $f(x) = \cos(\tan(x))$

f) $f(x) = \sec(\sec(x))$

g) $f(x) = \operatorname{arccot}(\operatorname{arccot}(x))$

h) $f(x) = \arctan(\ln(x))$

i) $f(x) = \arcsin(\operatorname{arccsc}(x))$

j) $f(x) = \arcsin(\operatorname{arcsec}(x))$

k) $f(x) = \arccos(\arcsin(x))$

l) $f(x) = \arccos(\operatorname{arcsec}(x))$

m) $f(x) = \cot(\sec(x))$

n) $f(x) = \operatorname{arcsec}\left(x^3\right)$

o) $f(x) = \cot(\tan(x))$

p) $f(x) = \tan\left(x^3\right)$

q) $f(x) = \cos(\sin(x))$

r) $f(x) = \cot(e^x)$

2. 求下列函数的导数：

a) $f(x) = \sec\left(e^{e^x}\right)$

b) $f(x) = \ln(\ln(\sec(x)))$

c) $f(x) = \tan(\cot(\ln(x)))$

d) $f(x) = \csc(\csc(\cos(x)))$

e) $f(x) = e^{\operatorname{arccsc}(\arcsin(x))}$

f) $f(x) = \arccos\left(\ln(x)^2\right)$

g) $f(x) = \operatorname{arccot}\left(\ln\left(x^2\right)\right)$

h) $f(x) = \operatorname{arccsc}(\operatorname{arccsc}(\operatorname{arccsc}(x)))$

i) $f(x) = e^{\cot(\csc(x))}$

j) $f(x) = \sin^3\left(x^3\right)$

k) $f(x) = \csc(\tan(\cos(x)))$

l) $f(x) = \operatorname{arcsec}(\cos(\tan(x)))$

3. 求下列函数的导数：

a) $f(x) = \cos(2e^x)$

b) $f(x) = \arcsin\left(\sqrt{x} + \operatorname{arccot}(x)\right)$

c) $f(x) = \arccos(e^x + \arctan(x))$

d) $f(x) = \tan(e^x + \cos(x))$

e) $f(x) = \operatorname{arccot}(\ln(x) + \cot(x))$

f) $f(x) = \cos(e^x + \cos(x))$

g) $f(x) = \arcsin(\cos(x)\cot(x))$

h) $f(x) = \cos(\sin(x)\cot(x))$

i) $f(x) = \tan\left(\cot^2(x)\right)$

j) $f(x) = \cot(\operatorname{arccot}(x)\arctan(x))$

k) $f(x) = \tan\left(\frac{\arccos(x)}{\operatorname{arccot}(x)}\right)$

l) $f(x) = \tan\left(\frac{\sqrt{x}}{\operatorname{arccot}(x)}\right)$

m) $f(x) = \sin\left(\frac{\operatorname{arccot}(x)}{\ln(x)}\right)$

n) $f(x) = \arcsin\left(\frac{e^x}{\arctan(x)}\right)$

4. 求下列函数的导数：

a) $f(x) = e^{x+\cot(x)}$

b) $f(x) = \ln\left(\ln(x) + \frac{1}{\sqrt{x^2+1}}\right)$

c) $f(x) = \ln\left(\operatorname{arccot}(x) + \arctan\left(\arccos(x)\right)\right)$

d) $f(x) = \tan\left(\ln\left(\arcsin(x)\right) + \arccos(x)\right)$

e) $f(x) = \operatorname{arccot}\left(\cos(x) + \tan\left(\cos(x)\right)\right)$

f) $f(x) = \arctan\left(e^{x} + \ln\left(\arctan(x)\right)\right)$

g) $f(x) = \cos\left(e^{\operatorname{arccot}(x)} \arccos(x)\right)$

h) $f(x) = \cos\left(\ln(x) \arcsin\left(\cos(x)\right)\right)$

i) $f(x) = \sin\left(\operatorname{arccot}\left(\sin(x)\right) \arcsin(x)\right)$

j) $f(x) = \arcsin\left(\sin(x) \arcsin\left(\arccos(x)\right)\right)$

## 答案

1.

a) $f'(x) = -\tan(\csc(x))\cot(x)\csc(x)\sec(\csc(x))$

b) $f'(x) = e^x e^{e^x}$

c) $f'(x) = \left(-\cot^2(e^x) - 1\right)e^x$

d) $f'(x) = \left(-\cot^2(x) - 1\right)\cos(\cot(x))$

e) $f'(x) = -\left(\tan^2(x) + 1\right)\sin(\tan(x))$

f) $f'(x) = \tan(x)\tan(\sec(x))\sec(x)\sec(\sec(x))$

g) $f'(x) = \dfrac{1}{(x^2+1)(\operatorname{arccot}^2(x)+1)}$

h) $f'(x) = \dfrac{1}{x\left(\ln(x)^2 + 1\right)}$

i) $f'(x) = -\dfrac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arccsc}^2(x)}}$

j) $f'(x) = \dfrac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arcsec}^2(x)}}$

k) $f'(x) = -\dfrac{1}{\sqrt{1-x^2}\sqrt{1-\arcsin^2(x)}}$

l) $f'(x) = -\dfrac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arcsec}^2(x)}}$

m) $f'(x) = \left(-\cot^2(\sec(x)) - 1\right)\tan(x)\sec(x)$

n) $f'(x) = \dfrac{3}{x^4\sqrt{1-\frac{1}{x^6}}}$

o) $f'(x) = \left(\tan^2(x)+1\right)\left(-\cot^2(\tan(x)) - 1\right)$

p) $f'(x) = 3x^2\left(\tan^2\left(x^3\right) + 1\right)$

q) $f'(x) = -\sin(\sin(x))\cos(x)$

r) $f'(x) = \left(-\cot^2(e^x) - 1\right)e^x$

2.

a) $f'(x) = e^x e^{e^x}\tan\left(e^{e^x}\right)\sec\left(e^{e^x}\right)$

b) $f'(x) = \dfrac{\tan(x)}{\ln(\sec(x))}$

c) $f'(x) = \dfrac{(\tan^2(\cot(\ln(x))) + 1)(-\cot^2(\ln(x)) - 1)}{x}$

d) $f'(x) = -\sin(x)\cot(\cos(x))\cot(\csc(\cos(x)))\csc(\cos(x))\csc(\csc(\cos(x)))$

e) $f'(x) = -\dfrac{e^{\operatorname{arccsc}(\arcsin(x))}}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arcsin^2(x)}}\arcsin^2(x)}$

f) $f'(x) = -\dfrac{2\ln(x)}{x\sqrt{1-\ln(x)^4}}$

g) $f'(x) = -\dfrac{2}{x\left(\ln(x^2)^2 + 1\right)}$

h) $f'(x) = -\dfrac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{\operatorname{arccsc}^2(x)}}\sqrt{1-\frac{1}{\operatorname{arccsc}^2(\operatorname{arccsc}(x))}}\operatorname{arccsc}^2(x)\operatorname{arccsc}^2(\operatorname{arccsc}(x))}$

i) $f'(x) = -\left(-\cot^2(\csc(x)) - 1\right) e^{\cot(\csc(x))} \cot(x) \csc(x)$

j) $f'(x) = 9x^2 \sin^2\left(x^3\right) \cos\left(x^3\right)$

k) $f'(x) = \left(\tan^2(\cos(x)) + 1\right) \sin(x) \cot(\tan(\cos(x))) \csc(\tan(\cos(x)))$

l) $f'(x) = -\dfrac{\left(\tan^2(x) + 1\right) \sin(\tan(x))}{\sqrt{1 - \frac{1}{\cos^2(\tan(x))}} \cos^2(\tan(x))}$

3.

a) $f'(x) = -2e^x \sin(2e^x)$

b) $f'(x) = \dfrac{-\frac{1}{x^2+1} + \frac{1}{2\sqrt{x}}}{\sqrt{1 - \left(\sqrt{x} + \operatorname{arccot}(x)\right)^2}}$

c) $f'(x) = -\dfrac{e^x + \frac{1}{x^2+1}}{\sqrt{1 - \left(e^x + \arctan(x)\right)^2}}$

d) $f'(x) = (e^x - \sin(x)) \left(\tan^2(e^x + \cos(x)) + 1\right)$

e) $f'(x) = -\dfrac{-\cot^2(x) - 1 + \frac{1}{x}}{\left(\ln(x) + \cot(x)\right)^2 + 1}$

f) $f'(x) = -(e^x - \sin(x)) \sin(e^x + \cos(x))$

g) $f'(x) = \dfrac{\left(-\cot^2(x) - 1\right)\cos(x) - \sin(x)\cot(x)}{\sqrt{-\cos^2(x)\cot^2(x) + 1}}$

h) $f'(x) = -\left(\left(-\cot^2(x) - 1\right)\sin(x) + \cos(x)\cot(x)\right) \sin(\sin(x)\cot(x))$

i) $f'(x) = \left(\tan^2\left(\cot^2(x)\right) + 1\right)\left(-2\cot^2(x) - 2\right)\cot(x)$

j) $f'(x) = \left(\dfrac{\operatorname{arccot}(x)}{x^2+1} - \dfrac{\arctan(x)}{x^2+1}\right)\left(-\cot^2(\operatorname{arccot}(x)\arctan(x)) - 1\right)$

k) $f'(x) = \left(\dfrac{\arccos(x)}{(x^2+1)\operatorname{arccot}^2(x)} - \dfrac{1}{\sqrt{1-x^2}\operatorname{arccot}(x)}\right)\left(\tan^2\left(\dfrac{\arccos(x)}{\operatorname{arccot}(x)}\right) + 1\right)$

l) $f'(x) = \left(\dfrac{\sqrt{x}}{(x^2+1)\operatorname{arccot}^2(x)} + \dfrac{1}{2\sqrt{x}\operatorname{arccot}(x)}\right)\left(\tan^2\left(\dfrac{\sqrt{x}}{\operatorname{arccot}(x)}\right) + 1\right)$

m) $f'(x) = \left(-\dfrac{1}{(x^2+1)\ln(x)} - \dfrac{\operatorname{arccot}(x)}{x\ln(x)^2}\right)\cos\left(\dfrac{\operatorname{arccot}(x)}{\ln(x)}\right)$

n) $f'(x) = \dfrac{\frac{e^x}{\arctan(x)} - \frac{e^x}{(x^2+1)\arctan^2(x)}}{\sqrt{-\frac{e^{2x}}{\arctan^2(x)} + 1}}$

4. a) $f'(x) = -e^{x+\cot(x)}\cot^2(x)$

b) $f'(x) = \dfrac{-\frac{x}{(x^2+1)^{\frac{3}{2}}} + \frac{1}{x}}{\ln(x) + \frac{1}{\sqrt{x^2+1}}}$

c) $f'(x) = \dfrac{-\frac{1}{x^2+1} - \frac{1}{\sqrt{1-x^2}(\arccos^2(x)+1)}}{\operatorname{arccot}(x) + \arctan(\arccos(x))}$

d) $f'(x) = \left(-\dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-x^2}\arcsin(x)}\right)\left(\tan^2(\ln(\arcsin(x)) + \arccos(x)) + 1\right)$

e) $f'(x) = -\dfrac{-(\tan^2(\cos(x)) + 1)\sin(x) - \sin(x)}{(\cos(x) + \tan(\cos(x)))^2 + 1}$

f) $f'(x) = \dfrac{e^x + \frac{1}{(x^2+1)\arctan(x)}}{(e^x + \ln(\arctan(x)))^2 + 1}$

g) $f'(x) = -\left(-\dfrac{e^{\operatorname{arccot}(x)}\arccos(x)}{x^2+1} - \dfrac{e^{\operatorname{arccot}(x)}}{\sqrt{1-x^2}}\right)\sin\left(e^{\operatorname{arccot}(x)}\arccos(x)\right)$

h) $f'(x) = -\left(-\dfrac{\ln(x)\sin(x)}{\sqrt{1-\cos^2(x)}} + \dfrac{\arcsin(\cos(x))}{x}\right)\sin(\ln(x)\arcsin(\cos(x)))$

i) $f'(x) = \left(-\dfrac{\cos(x)\arcsin(x)}{\sin^2(x)+1} + \dfrac{\operatorname{arccot}(\sin(x))}{\sqrt{1-x^2}}\right)\cos(\operatorname{arccot}(\sin(x))\arcsin(x))$

j) $f'(x) = \dfrac{\cos(x)\arcsin(\arccos(x)) - \frac{\sin(x)}{\sqrt{1-x^2}\sqrt{1-\arccos^2(x)}}}{\sqrt{-\sin^2(x)\arcsin^2(\arccos(x)) + 1}}$