复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = e^{\csc(x)}$

b) $f(x) = \tan(\sin(x))$

c) $f(x) = \sec(\ln(x))$

d) $f(x) = \csc(\tan(x))$

e) $f(x) = \tan(e^x)$

f) $f(x) = \cot(\sec(x))$

g) $f(x) = \operatorname{arccot}(\arcsin(x))$

h) $f(x) = \arcsin(\arctan(x))$

i) $f(x) = \arccos(e^x)$

j) $f(x) = \ln(\operatorname{arcsec}(x))$

k) $f(x) = \arctan(\operatorname{arccsc}(x))$

l) $f(x) = \operatorname{arccsc}(\arccos(x))$

m) $f(x) = \operatorname{arcsec}\left(x^3\right)$

n) $f(x) = \arcsin(\sec(x))$

o) $f(x) = \operatorname{arccsc}(\arccos(x))$

p) $f(x) = \cot(e^x)$

q) $f(x) = e^{\operatorname{arccot}(x)}$

r) $f(x) = \tan(\cot(x))$

2. 求下列函数的导数:

a) $f(x) = \csc(\ln(\ln(x)))$

b) $f(x) = \sec(\sin(\sec(x)))$

c) $f(x) = \sqrt{\csc(\tan(x))}$

d) $f(x) = e^{\tan(\tan(x))}$

e) $f(x) = \operatorname{arccos}(\ln(\arctan(x)))$

f) $f(x) = \operatorname{arcsec}(\arctan(e^x))$

g) $f(x) = \operatorname{arccsc}\left(\operatorname{arccot}^2(x)\right)$

h) $f(x) = \operatorname{arcsec}^2(\arccos(x))$

i) $f(x) = \sqrt{1 - \sec^2(x)}$

j) $f(x) = \ln(\cos(\ln(x)))$

k) $f(x) = \cos(\csc(\cos(x)))$

l) $f(x) = \csc(\cot(\sin(x)))$

3. 求下列函数的导数:

a) $f(x) = e^{e^x + \tan(x)}$

b) $f(x) = \operatorname{arccot}(\cos(x) + \arcsin(x))$

c) $f(x) = \ln(e^x + \arcsin(x))$

d) $f(x) = \sqrt{\cot(x) + \arcsin(x)}$

e) $f(x) = \cot\left(\arccos\left(x\right) + \arcsin\left(x\right)\right)$

f) $f(x) = \tan\left(2\sin\left(x\right)\right)$

g) $f(x) = \tan\left(\sqrt{x}\arccos\left(x\right)\right)$

h) $f(x) = \ln\left(\ln\left(x\right)^2\right)$

i) $f(x) = \arctan\left(\sqrt{x}\arccos\left(x\right)\right)$

j) $f(x) = \arccos\left(\sin\left(x\right)\cot\left(x\right)\right)$

k) $f(x) = \sqrt{\dfrac{\arccos\left(x\right)}{\operatorname{arccot}\left(x\right)}}$

l) $f(x) = \arcsin\left(\sqrt{x}e^{-x}\right)$

m) $f(x) = \arccos\left(e^{-x}\arccos\left(x\right)\right)$

n) $f(x) = \sin\left(\dfrac{\cos\left(x\right)}{\operatorname{arccot}\left(x\right)}\right)$

4. 求下列函数的导数：

a) $f(x) = \arcsin\left(\sqrt{\tan\left(x\right)} + \arccos\left(x\right)\right)$

b) $f(x) = \cos\left(\ln\left(x\right) + \arcsin\left(e^x\right)\right)$

c) $f(x) = \ln\left(\sqrt{1-x^2} + \arctan(x)\right)$

d) $f(x) = \sin(\cos(x) + \arcsin(\operatorname{arccot}(x)))$

e) $f(x) = e^{e^x + \cot(\sin(x))}$

f) $f(x) = \arccos\left(\tan(x) + \frac{\sqrt{1-x^2}}{x}\right)$

g) $f(x) = \arccos\left(\sqrt[4]{x}\arccos(x)\right)$

h) $f(x) = \arcsin\left(e^{\operatorname{arccot}(x)}\arccos(x)\right)$

i) $f(x) = \tan\left(e^{\cot(x)}\tan(x)\right)$

j) $f(x) = \cos\left(\frac{\operatorname{arccot}(x)}{x}\right)$

答案

1.

a) $f'(x) = -e^{\csc(x)} \cot(x) \csc(x)$

b) $f'(x) = \left(\tan^2(\sin(x)) + 1\right) \cos(x)$

c) $f'(x) = \frac{\tan(\ln(x)) \sec(\ln(x))}{x}$

d) $f'(x) = -\left(\tan^2(x) + 1\right) \cot(\tan(x)) \csc(\tan(x))$

e) $f'(x) = \left(\tan^2(e^x) + 1\right) e^x$

f) $f'(x) = \left(-\cot^2(\sec(x)) - 1\right) \tan(x) \sec(x)$

g) $f'(x) = -\frac{1}{\sqrt{1 - x^2}\left(\arcsin^2(x) + 1\right)}$

h) $f'(x) = \frac{1}{\sqrt{1 - \arctan^2(x)}\,(x^2 + 1)}$

i) $f'(x) = -\frac{e^x}{\sqrt{1 - e^{2x}}}$

j) $f'(x) = \frac{1}{x^2\sqrt{1 - \frac{1}{x^2}}\,\operatorname{arcsec}(x)}$

k) $f'(x) = -\frac{1}{x^2\sqrt{1 - \frac{1}{x^2}}\,(\operatorname{arccsc}^2(x) + 1)}$

l) $f'(x) = \frac{1}{\sqrt{1 - x^2}\sqrt{1 - \frac{1}{\arccos^2(x)}}\,\arccos^2(x)}$

m) $f'(x) = \frac{3}{x^4\sqrt{1 - \frac{1}{x^6}}}$

n) $f'(x) = \frac{\tan(x)\sec(x)}{\sqrt{1 - \sec^2(x)}}$

o) $f'(x) = \frac{1}{\sqrt{1 - x^2}\sqrt{1 - \frac{1}{\arccos^2(x)}}\,\arccos^2(x)}$

p) $f'(x) = \left(-\cot^2(e^x) - 1\right) e^x$

q) $f'(x) = -\frac{e^{\operatorname{arccot}(x)}}{x^2 + 1}$

r) $f'(x) = \left(\tan^2(\cot(x)) + 1\right)\left(-\cot^2(x) - 1\right)$

2.

a) $f'(x) = -\frac{\cot(\ln(\ln(x))) \csc(\ln(\ln(x)))}{x \ln(x)}$

b) $f'(x) = \cos(\sec(x)) \tan(x) \tan(\sin(\sec(x))) \sec(x) \sec(\sin(\sec(x)))$

c) $f'(x) = -\frac{(\tan^2(x) + 1) \cot(\tan(x)) \sqrt{\csc(\tan(x))}}{2}$

d) $f'(x) = \left(\tan^2(x) + 1\right)\left(\tan^2(\tan(x)) + 1\right) e^{\tan(\tan(x))}$

e) $f'(x) = -\frac{1}{\sqrt{1 - \ln(\arctan(x))^2}\,(x^2 + 1)\arctan(x)}$

f) $f'(x) = \frac{e^x}{\sqrt{1 - \frac{1}{\arctan^2(e^x)}}\,(e^{2x} + 1)\arctan^2(e^x)}$

g) $f'(x) = \frac{2}{\sqrt{1 - \frac{1}{\operatorname{arccot}^4(x)}}\,(x^2 + 1)\operatorname{arccot}^3(x)}$

h) $f'(x) = -\dfrac{2\operatorname{arcsec}(\arccos(x))}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arccos^2(x)}}\arccos^2(x)}$

i) $f'(x) = -\dfrac{\tan(x)\sec^2(x)}{\sqrt{1-\sec^2(x)}}$

j) $f'(x) = -\dfrac{\sin(\ln(x))}{x\cos(\ln(x))}$

k) $f'(x) = -\sin(x)\sin(\csc(\cos(x)))\cot(\cos(x))\csc(\cos(x))$

l) $f'(x) = -\left(-\cot^2(\sin(x))-1\right)\cos(x)\cot(\cot(\sin(x)))\csc(\cot(\sin(x)))$

3. a) $f'(x) = \left(e^x+\tan^2(x)+1\right)e^{e^x+\tan(x)}$

b) $f'(x) = -\dfrac{-\sin(x)+\frac{1}{\sqrt{1-x^2}}}{(\cos(x)+\arcsin(x))^2+1}$

c) $f'(x) = \dfrac{e^x+\frac{1}{\sqrt{1-x^2}}}{e^x+\arcsin(x)}$

d) $f'(x) = \dfrac{-\frac{\cot^2(x)}{2}-\frac{1}{2}+\frac{1}{2\sqrt{1-x^2}}}{\sqrt{\cot(x)+\arcsin(x)}}$

e) $f'(x) = 0$

f) $f'(x) = 2\left(\tan^2(2\sin(x))+1\right)\cos(x)$

g) $f'(x) = \left(-\dfrac{\sqrt{x}}{\sqrt{1-x^2}}+\dfrac{\arccos(x)}{2\sqrt{x}}\right)\left(\tan^2\left(\sqrt{x}\arccos(x)\right)+1\right)$

h) $f'(x) = \dfrac{2}{x\ln(x)}$

i) $f'(x) = \dfrac{-\frac{\sqrt{x}}{\sqrt{1-x^2}}+\frac{\arccos(x)}{2\sqrt{x}}}{x\arccos^2(x)+1}$

j) $f'(x) = -\dfrac{(-\cot^2(x)-1)\sin(x)+\cos(x)\cot(x)}{\sqrt{-\sin^2(x)\cot^2(x)+1}}$

k) $f'(x) = \dfrac{\sqrt{\frac{\arccos(x)}{\operatorname{arccot}(x)}}\left(\frac{\arccos(x)}{2(x^2+1)\operatorname{arccot}^2(x)}-\frac{1}{2\sqrt{1-x^2}\operatorname{arccot}(x)}\right)\operatorname{arccot}(x)}{\arccos(x)}$

l) $f'(x) = \dfrac{-\sqrt{x}e^{-x}+\frac{e^{-x}}{2\sqrt{x}}}{\sqrt{-xe^{-2x}+1}}$

m) $f'(x) = -\dfrac{-e^{-x}\arccos(x)-\frac{e^{-x}}{\sqrt{1-x^2}}}{\sqrt{1-e^{-2x}\arccos^2(x)}}$

n) $f'(x) = \left(-\dfrac{\sin(x)}{\operatorname{arccot}(x)}+\dfrac{\cos(x)}{(x^2+1)\operatorname{arccot}^2(x)}\right)\cos\left(\dfrac{\cos(x)}{\operatorname{arccot}(x)}\right)$

4.

a) $f'(x) = \dfrac{\frac{\frac{\tan^2(x)}{2}+\frac{1}{2}}{\sqrt{\tan(x)}} - \frac{1}{\sqrt{1-x^2}}}{\sqrt{1-\left(\sqrt{\tan(x)}+\arccos(x)\right)^2}}$

b) $f'(x) = -\left(\dfrac{e^x}{\sqrt{1-e^{2x}}} + \dfrac{1}{x}\right)\sin\left(\ln(x)+\arcsin(e^x)\right)$

c) $f'(x) = \dfrac{-\frac{x}{\sqrt{1-x^2}} + \frac{1}{x^2+1}}{\sqrt{1-x^2}+\arctan(x)}$

d) $f'(x) = \left(-\sin(x) - \dfrac{1}{\sqrt{1-\operatorname{arccot}^2(x)}\,(x^2+1)}\right)\cos\left(\cos(x)+\arcsin(\operatorname{arccot}(x))\right)$

e) $f'(x) = \left(\left(-\cot^2(\sin(x))-1\right)\cos(x)+e^x\right)e^{e^x+\cot(\sin(x))}$

f) $f'(x) = -\dfrac{\tan^2(x)+1-\frac{1}{\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}}{x^2}}{\sqrt{1-\left(\tan(x)+\frac{\sqrt{1-x^2}}{x}\right)^2}}$

g) $f'(x) = -\dfrac{-\frac{\sqrt[4]{x}}{\sqrt{1-x^2}} + \frac{\arccos(x)}{4x^{\frac{3}{4}}}}{\sqrt{-\sqrt{x}\arccos^2(x)+1}}$

h) $f'(x) = \dfrac{-\frac{e^{\operatorname{arccot}(x)}\arccos(x)}{x^2+1} - \frac{e^{\operatorname{arccot}(x)}}{\sqrt{1-x^2}}}{\sqrt{-e^{2\operatorname{arccot}(x)}\arccos^2(x)+1}}$

i) $f'(x) = \left(\left(\tan^2(x)+1\right)e^{\cot(x)} + \left(-\cot^2(x)-1\right)e^{\cot(x)}\tan(x)\right)\left(\tan^2\left(e^{\cot(x)}\tan(x)\right)+1\right)$

j) $f'(x) = -\left(-\dfrac{1}{x(x^2+1)} - \dfrac{\operatorname{arccot}(x)}{x^2}\right)\sin\left(\dfrac{\operatorname{arccot}(x)}{x}\right)$