复合函数的求导法则

练习

1. 求下列函数的导数:

a) $f(x) = e^{\csc(x)}$

b) $f(x) = \ln(\tan(x))$

c) $f(x) = \tan(\sin(x))$

d) $f(x) = \sqrt{\sin(x)}$

e) $f(x) = \sqrt{\cos(x)}$

f) $f(x) = \sin(\sin(x))$

g) $f(x) = \arcsin(\arccos(x))$

h) $f(x) = \operatorname{arccsc}(\arccos(x))$

i) $f(x) = \arcsin(\operatorname{arccot}(x))$

j) $f(x) = \operatorname{arccsc}(\operatorname{arcsec}(x))$

k) $f(x) = \arcsin(\arcsin(x))$

l) $f(x) = \arcsin^2(x)$

m) $f(x) = \operatorname{arccsc}\left(x^3\right)$

n) $f(x) = \cos(\csc(x))$

o) $f(x) = \arccos(\cot(x))$

p) $f(x) = \operatorname{arccot}(\arctan(x))$

q) $f(x) = \cos\left(x^3\right)$

r) $f(x) = \cos(\tan(x))$

2. 求下列函数的导数：

a) $f(x) = \sec(\sin(\sin(x)))$

b) $f(x) = e^{\sin(\cos(x))}$

c) $f(x) = \cos(\cos(\tan(x)))$

d) $f(x) = \tan(\csc(\ln(x)))$

e) $f(x) = \operatorname{arccot}(\arccos(\arccos(x)))$

f) $f(x) = \operatorname{arccot}(\arccos(\arcsin(x)))$

g) $f(x) = \arctan^2(\operatorname{arcsec}(x))$

h) $f(x) = \arcsin(x)$

i) $f(x) = \cot\left(x\sqrt{1+\frac{1}{x^2}}\right)$

j) $f(x) = \ln(\arctan(\cot(x)))$

k) $f(x) = \operatorname{arccsc}\left(\ln\left(x^3\right)\right)$

l) $f(x) = \cot\left(\sec\left(x^3\right)\right)$

3. 求下列函数的导数:

a) $f(x) = \ln(e^x + \cot(x))$

b) $f(x) = \sin\left(\sqrt{x} + \sin(x)\right)$

c) $f(x) = \sqrt{\arccos(x) + \arctan(x)}$

d) $f(x) = \cot(\ln(x) + \cot(x))$

e) $f(x) = \arctan(\operatorname{arccot}(x) + \arcsin(x))$

f) $f(x) = \cos(\tan(x) + \cot(x))$

g) $f(x) = e^{\ln(x)\tan(x)}$

h) $f(x) = \cos(\tan(x)\arctan(x))$

i) $f(x) = \tan(e^x \tan(x))$

j) $f(x) = \sin(\tan(x)\operatorname{arccot}(x))$

k) $f(x) = \cos\left(\frac{\sin(x)}{\arccos(x)}\right)$

l) $f(x) = \tan\left(\frac{\ln(x)}{\arcsin(x)}\right)$

m) $f(x) = \operatorname{arccot}\left(\frac{\sqrt{x}}{\cos(x)}\right)$

n) $f(x) = \cot\left(\frac{\cot(x)}{\arccos(x)}\right)$

4. 求下列函数的导数:

a) $f(x) = e^{\cos(x)}\arccos(x)$

b) $f(x) = \arcsin\left(\frac{x}{\sqrt{x^2+1}} + \arccos(x)\right)$

c) $f(x) = \sqrt{\sin\left(\sqrt{x}\right) + \sin(x)}$

d) $f(x) = \ln\left(e^x + \cos\left(e^x\right)\right)$

e) $f(x) = \sin\left(\sqrt{\arcsin(x)} + \arcsin(x)\right)$

f) $f(x) = e^{\arctan(x) + \arctan(\ln(x))}$

g) $f(x) = \arccos\left(\ln\left(\ln(x)\right)\sin(x)\right)$

h) $f(x) = \sin\left(\arcsin\left(\cos(x)\right)\arctan(x)\right)$

i) $f(x) = \arccos\left(\sqrt{x}\arccos\left(\tan(x)\right)\right)$

j) $f(x) = \cos\left(\sin(x)\operatorname{arccot}\left(\sqrt{x}\right)\right)$

答案

1.

a) $f'(x) = -e^{\csc(x)} \cot(x) \csc(x)$

b) $f'(x) = \dfrac{\tan^2(x) + 1}{\tan(x)}$

c) $f'(x) = \left(\tan^2(\sin(x)) + 1\right) \cos(x)$

d) $f'(x) = \dfrac{\cos(x)}{2\sqrt{\sin(x)}}$

e) $f'(x) = -\dfrac{\sin(x)}{2\sqrt{\cos(x)}}$

f) $f'(x) = \cos(x)\cos(\sin(x))$

g) $f'(x) = -\dfrac{1}{\sqrt{1-x^2}\sqrt{1-\arccos^2(x)}}$

h) $f'(x) = \dfrac{1}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arccos^2(x)}}\arccos^2(x)}$

i) $f'(x) = -\dfrac{1}{\sqrt{1-\operatorname{arccot}^2(x)}\,(x^2+1)}$

j) $f'(x) = -\dfrac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{1}{\operatorname{arcsec}^2(x)}}\operatorname{arcsec}^2(x)}$

k) $f'(x) = \dfrac{1}{\sqrt{1-x^2}\sqrt{1-\arcsin^2(x)}}$

l) $f'(x) = \dfrac{2\arcsin(x)}{\sqrt{1-x^2}}$

m) $f'(x) = -\dfrac{3}{x^4\sqrt{1-\frac{1}{x^6}}}$

n) $f'(x) = \sin(\csc(x))\cot(x)\csc(x)$

o) $f'(x) = -\dfrac{-\cot^2(x) - 1}{\sqrt{1-\cot^2(x)}}$

p) $f'(x) = -\dfrac{1}{(x^2+1)(\arctan^2(x)+1)}$

q) $f'(x) = -3x^2\sin\left(x^3\right)$

r) $f'(x) = -\left(\tan^2(x)+1\right)\sin(\tan(x))$

2.

a) $f'(x) = \cos(x)\cos(\sin(x))\tan(\sin(\sin(x)))\sec(\sin(\sin(x)))$

b) $f'(x) = -e^{\sin(\cos(x))}\sin(x)\cos(\cos(x))$

c) $f'(x) = \left(\tan^2(x)+1\right)\sin(\cos(\tan(x)))\sin(\tan(x))$

d) $f'(x) = -\dfrac{(\tan^2(\csc(\ln(x)))+1)\cot(\ln(x))\csc(\ln(x))}{x}$

e) $f'(x) = -\dfrac{1}{\sqrt{1-x^2}\sqrt{1-\arccos^2(x)}\,(\arccos^2(\arccos(x))+1)}$

f) $f'(x) = \dfrac{1}{\sqrt{1-x^2}\sqrt{1-\arcsin^2(x)}\,(\arccos^2(\arcsin(x))+1)}$

g) $f'(x) = \dfrac{2\arctan(\operatorname{arcsec}(x))}{x^2\sqrt{1-\frac{1}{x^2}}\,(\operatorname{arcsec}^2(x)+1)}$

h) $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

i) $f'(x) = \left(\sqrt{1+\dfrac{1}{x^2}} - \dfrac{1}{x^2\sqrt{1+\frac{1}{x^2}}}\right)\left(-\cot^2\left(x\sqrt{1+\dfrac{1}{x^2}}\right)-1\right)$

j) $f'(x) = \dfrac{-\cot^2(x)-1}{(\cot^2(x)+1)\arctan(\cot(x))}$

k) $f'(x) = -\dfrac{3}{x\sqrt{1-\frac{1}{\ln(x^3)^2}}\ln(x^3)^2}$

l) $f'(x) = 3x^2\left(-\cot^2\left(\sec\left(x^3\right)\right)-1\right)\tan\left(x^3\right)\sec\left(x^3\right)$

3.

a) $f'(x) = \dfrac{e^x-\cot^2(x)-1}{e^x+\cot(x)}$

b) $f'(x) = \left(\cos(x)+\dfrac{1}{2\sqrt{x}}\right)\cos\left(\sqrt{x}+\sin(x)\right)$

c) $f'(x) = \dfrac{\frac{1}{2(x^2+1)} - \frac{1}{2\sqrt{1-x^2}}}{\sqrt{\arccos(x)+\arctan(x)}}$

d) $f'(x) = \left(-\cot^2(\ln(x)+\cot(x))-1\right)\left(-\cot^2(x)-1+\dfrac{1}{x}\right)$

e) $f'(x) = \dfrac{-\frac{1}{x^2+1}+\frac{1}{\sqrt{1-x^2}}}{(\operatorname{arccot}(x)+\arcsin(x))^2+1}$

f) $f'(x) = -\left(\tan^2(x)-\cot^2(x)\right)\sin(\tan(x)+\cot(x))$

g) $f'(x) = \left(\left(\tan^2(x)+1\right)\ln(x)+\dfrac{\tan(x)}{x}\right)e^{\ln(x)\tan(x)}$

h) $f'(x) = -\left(\left(\tan^2(x)+1\right)\arctan(x)+\dfrac{\tan(x)}{x^2+1}\right)\sin(\tan(x)\arctan(x))$

i) $f'(x) = \left(\left(\tan^2(x)+1\right)e^x+e^x\tan(x)\right)\left(\tan^2(e^x\tan(x))+1\right)$

j) $f'(x) = \left(\left(\tan^2(x)+1\right)\operatorname{arccot}(x)-\dfrac{\tan(x)}{x^2+1}\right)\cos(\tan(x)\operatorname{arccot}(x))$

k) $f'(x) = -\left(\dfrac{\cos(x)}{\arccos(x)}+\dfrac{\sin(x)}{\sqrt{1-x^2}\arccos^2(x)}\right)\sin\left(\dfrac{\sin(x)}{\arccos(x)}\right)$

l) $f'(x) = \left(-\dfrac{\ln(x)}{\sqrt{1-x^2}\arcsin^2(x)}+\dfrac{1}{x\arcsin(x)}\right)\left(\tan^2\left(\dfrac{\ln(x)}{\arcsin(x)}\right)+1\right)$

m) $f'(x) = -\frac{\frac{\sqrt{x}\sin(x)}{\cos^2(x)} + \frac{1}{2\sqrt{x}\cos(x)}}{\frac{x}{\cos^2(x)} + 1}$

n) $f'(x) = \left(\frac{-\cot^2(x) - 1}{\arccos(x)} + \frac{\cot(x)}{\sqrt{1-x^2}\arccos^2(x)}\right)\left(-\cot^2\left(\frac{\cot(x)}{\arccos(x)}\right) - 1\right)$

4.

a) $f'(x) = -e^{\cos(x)}\sin(x)\arccos(x) - \frac{e^{\cos(x)}}{\sqrt{1-x^2}}$

b) $f'(x) = \frac{-\frac{x^2}{(x^2+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{x^2+1}} - \frac{1}{\sqrt{1-x^2}}}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+1}} + \arccos(x)\right)^2}}$

c) $f'(x) = \frac{\frac{\cos(x)}{2} + \frac{\cos(\sqrt{x})}{4\sqrt{x}}}{\sqrt{\sin(\sqrt{x}) + \sin(x)}}$

d) $f'(x) = \frac{-e^x\sin(e^x) + e^x}{e^x + \cos(e^x)}$

e) $f'(x) = \left(\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}\sqrt{\arcsin(x)}}\right)\cos\left(\sqrt{\arcsin(x)} + \arcsin(x)\right)$

f) $f'(x) = \left(\frac{1}{x^2+1} + \frac{1}{x\left(\ln(x)^2+1\right)}\right)e^{\arctan(x)+\arctan(\ln(x))}$

g) $f'(x) = -\frac{\ln(\ln(x))\cos(x) + \frac{\sin(x)}{x\ln(x)}}{\sqrt{-\ln(\ln(x))^2\sin^2(x) + 1}}$

h) $f'(x) = \left(\frac{\arcsin(\cos(x))}{x^2+1} - \frac{\sin(x)\arctan(x)}{\sqrt{1-\cos^2(x)}}\right)\cos(\arcsin(\cos(x))\arctan(x))$

i) $f'(x) = -\frac{-\frac{\sqrt{x}\left(\tan^2(x)+1\right)}{\sqrt{1-\tan^2(x)}} + \frac{\arccos(\tan(x))}{2\sqrt{x}}}{\sqrt{-x\arccos^2(\tan(x)) + 1}}$

j) $f'(x) = -\left(\cos(x)\operatorname{arccot}(\sqrt{x}) - \frac{\sin(x)}{2\sqrt{x}(x+1)}\right)\sin\left(\sin(x)\operatorname{arccot}(\sqrt{x})\right)$