复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = \sqrt{\ln(x)}$

b) $f(x) = \ln(e^x)$

c) $f(x) = \sqrt{\sec(x)}$

d) $f(x) = e^{\cos(x)}$

e) $f(x) = \ln\left(\sqrt{x}\right)$

f) $f(x) = \csc(\cot(x))$

g) $f(x) = e^{x^2}$

h) $f(x) = \arccos(\arccos(x))$

i) $f(x) = \arcsin(\operatorname{arcsec}(x))$

j) $f(x) = \arcsin(\arctan(x))$

k) $f(x) = \operatorname{arccsc}(\arcsin(x))$

l) $f(x) = \arccos(e^x)$

m) $f(x) = \tan(\cos(x))$

n) $f(x) = e^{3x}$

o) $f(x) = \sec(\csc(x))$

p) $f(x) = \sin^3(x)$

q) $f(x) = \ln(x)^3$

r) $f(x) = \sin\left(x^3\right)$

2. 求下列函数的导数：

a) $f(x) = \sqrt{\csc(\cot(x))}$

b) $f(x) = \cot(\csc(\csc(x)))$

c) $f(x) = \cot(\tan(\sec(x)))$

d) $f(x) = \tan\left(\sqrt{\sin(x)}\right)$

e) $f(x) = \ln(\operatorname{arccot}(\operatorname{arccsc}(x)))$

f) $f(x) = e^{2\arccos(x)}$

g) $f(x) = \operatorname{arcsec}(\arctan(\arccos(x)))$

h) $f(x) = \arctan^2(\operatorname{arccsc}(x))$

i) $f(x) = \sec(\csc(\csc(x)))$

j) $f(x) = \operatorname{arccot}\left(\frac{\sqrt{x^2+1}}{x}\right)$

k) $f(x) = \cos\left(e^{\arccos(x)}\right)$

l) $f(x) = \tan(\csc(\sin(x)))$

3. 求下列函数的导数：

a) $f(x) = e^{\sin(x)+\arctan(x)}$

b) $f(x) = \tan(e^x + \arccos(x))$

c) $f(x) = \operatorname{arccot}\left(\sqrt{x} + \arcsin(x)\right)$

d) $f(x) = \ln\left(\sqrt{x} + \cot(x)\right)$

e) $f(x) = \sin\left(e^x + \cot(x)\right)$

f) $f(x) = \operatorname{arccot}\left(\cos(x) + \operatorname{arccot}(x)\right)$

g) $f(x) = \cot\left(\ln(x)\tan(x)\right)$

h) $f(x) = \cos\left(e^x \operatorname{arccot}(x)\right)$

i) $f(x) = \cos\left(\sqrt{x}\cos(x)\right)$

j) $f(x) = \tan\left(\ln(x)\sin(x)\right)$

k) $f(x) = \sqrt{\dfrac{\sin(x)}{\cos(x)}}$

l) $f(x) = \sqrt{\dfrac{\cos(x)}{\sqrt{x}}}$

m) $f(x) = \arcsin\left(\dfrac{\cot(x)}{\sqrt{x}}\right)$

n) $f(x) = \sqrt{\dfrac{\arcsin(x)}{\tan(x)}}$

4. 求下列函数的导数：

a) $f(x) = \cos\left(\cos(x) + \tan\left(\sqrt{x}\right)\right)$

b) $f(x) = \arcsin\left(\cot(x) + \dfrac{\sqrt{1-x^2}}{x}\right)$

c) $f(x) = \cot\left(\frac{x}{\sqrt{1-x^2}} + \arccos(x)\right)$

d) $f(x) = \cos\left(\operatorname{arccot}(x) + \frac{\sqrt{1-x^2}}{x}\right)$

e) $f(x) = \sin\left(e^{\sqrt{x}} + \tan(x)\right)$

f) $f(x) = \arccos(\sin(\ln(x)) + \arccos(x))$

g) $f(x) = \cos\left(\sqrt{x}\operatorname{arccot}(\arcsin(x))\right)$

h) $f(x) = \arcsin(\cot(\sin(x))\arctan(x))$

i) $f(x) = \tan(\sin(\ln(x))\arctan(x))$

j) $f(x) = \arctan(\arcsin(x)\arctan(\arcsin(x)))$

答案

1.

a) $f'(x) = \frac{1}{2x\sqrt{\ln(x)}}$

b) $f'(x) = 1$

c) $f'(x) = \frac{\tan(x)\sqrt{\sec(x)}}{2}$

d) $f'(x) = -e^{\cos(x)}\sin(x)$

e) $f'(x) = \frac{1}{2x}$

f) $f'(x) = -\left(-\cot^2(x) - 1\right)\cot(\cot(x))\csc(\cot(x))$

g) $f'(x) = 2xe^{x^2}$

h) $f'(x) = \frac{1}{\sqrt{1-x^2}\sqrt{1-\arccos^2(x)}}$

i) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arcsec}^2(x)}}$

j) $f'(x) = \frac{1}{\sqrt{1-\arctan^2(x)}\,(x^2+1)}$

k) $f'(x) = -\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arcsin^2(x)}}\arcsin^2(x)}$

l) $f'(x) = -\frac{e^x}{\sqrt{1-e^{2x}}}$

m) $f'(x) = -\left(\tan^2(\cos(x)) + 1\right)\sin(x)$

n) $f'(x) = 3e^{3x}$

o) $f'(x) = -\tan(\csc(x))\cot(x)\csc(x)\sec(\csc(x))$

p) $f'(x) = 3\sin^2(x)\cos(x)$

q) $f'(x) = \frac{3\ln(x)^2}{x}$

r) $f'(x) = 3x^2\cos\left(x^3\right)$

2.

a) $f'(x) = -\frac{(-\cot^2(x)-1)\cot(\cot(x))\sqrt{\csc(\cot(x))}}{2}$

b) $f'(x) = \left(-\cot^2(\csc(\csc(x))) - 1\right)\cot(x)\cot(\csc(x))\csc(x)\csc(\csc(x))$

c) $f'(x) = \left(\tan^2(\sec(x)) + 1\right)\left(-\cot^2(\tan(\sec(x))) - 1\right)\tan(x)\sec(x)$

d) $f'(x) = \frac{\left(\tan^2\left(\sqrt{\sin(x)}\right) + 1\right)\cos(x)}{2\sqrt{\sin(x)}}$

e) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\,(\operatorname{arccsc}^2(x)+1)\operatorname{arccot}(\operatorname{arccsc}(x))}$

f) $f'(x) = -\frac{2e^{2\arccos(x)}}{\sqrt{1-x^2}}$

g) $f'(x) = -\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arctan^2(\arccos(x))}}\,(\arccos^2(x)+1)\arctan^2(\arccos(x))}$

h) $f'(x) = -\frac{2\arctan\left(\operatorname{arccsc}(x)\right)}{x^2\sqrt{1-\frac{1}{x^2}}\left(\operatorname{arccsc}^2(x)+1\right)}$

i) $f'(x) = \tan\left(\csc\left(\csc(x)\right)\right)\cot(x)\cot\left(\csc(x)\right)\csc(x)\csc\left(\csc(x)\right)\sec\left(\csc\left(\csc(x)\right)\right)$

j) $f'(x) = -\frac{\frac{1}{\sqrt{x^2+1}} - \frac{\sqrt{x^2+1}}{x^2}}{1+\frac{x^2+1}{x^2}}$

k) $f'(x) = \frac{e^{\arccos(x)}\sin\left(e^{\arccos(x)}\right)}{\sqrt{1-x^2}}$

l) $f'(x) = -\left(\tan^2\left(\csc\left(\sin(x)\right)\right)+1\right)\cos(x)\cot\left(\sin(x)\right)\csc\left(\sin(x)\right)$

3.

a) $f'(x) = \left(\cos(x) + \frac{1}{x^2+1}\right)e^{\sin(x)+\arctan(x)}$

b) $f'(x) = \left(e^x - \frac{1}{\sqrt{1-x^2}}\right)\left(\tan^2\left(e^x+\arccos(x)\right)+1\right)$

c) $f'(x) = -\frac{\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}}}{\left(\sqrt{x}+\arcsin(x)\right)^2+1}$

d) $f'(x) = \frac{-\cot^2(x) - 1 + \frac{1}{2\sqrt{x}}}{\sqrt{x}+\cot(x)}$

e) $f'(x) = \left(e^x - \cot^2(x) - 1\right)\cos\left(e^x+\cot(x)\right)$

f) $f'(x) = -\frac{-\sin(x) - \frac{1}{x^2+1}}{\left(\cos(x)+\operatorname{arccot}(x)\right)^2+1}$

g) $f'(x) = \left(\left(\tan^2(x)+1\right)\ln(x) + \frac{\tan(x)}{x}\right)\left(-\cot^2\left(\ln(x)\tan(x)\right)-1\right)$

h) $f'(x) = -\left(e^x\operatorname{arccot}(x) - \frac{e^x}{x^2+1}\right)\sin\left(e^x\operatorname{arccot}(x)\right)$

i) $f'(x) = -\left(-\sqrt{x}\sin(x) + \frac{\cos(x)}{2\sqrt{x}}\right)\sin\left(\sqrt{x}\cos(x)\right)$

j) $f'(x) = \left(\ln(x)\cos(x) + \frac{\sin(x)}{x}\right)\left(\tan^2\left(\ln(x)\sin(x)\right)+1\right)$

k) $f'(x) = \frac{\sqrt{\frac{\sin(x)}{\cos(x)}}\left(\frac{\sin^2(x)}{2\cos^2(x)}+\frac{1}{2}\right)\cos(x)}{\sin(x)}$

l) $f'(x) = \frac{\sqrt{x}\sqrt{\frac{\cos(x)}{\sqrt{x}}}\left(-\frac{\sin(x)}{2\sqrt{x}} - \frac{\cos(x)}{4x^{\frac{3}{2}}}\right)}{\cos(x)}$

m) $f'(x) = \frac{\frac{-\cot^2(x)-1}{\sqrt{x}} - \frac{\cot(x)}{2x^{\frac{3}{2}}}}{\sqrt{1-\frac{\cot^2(x)}{x}}}$

n) $f'(x) = \frac{\sqrt{\frac{\arcsin(x)}{\tan(x)}}\left(\frac{\left(-\tan^2(x)-1\right)\arcsin(x)}{2\tan^2(x)} + \frac{1}{2\sqrt{1-x^2}\tan(x)}\right)\tan(x)}{\arcsin(x)}$

4.

a) $f'(x) = -\left(-\sin(x) + \frac{\tan^2(\sqrt{x})+1}{2\sqrt{x}}\right)\sin\left(\cos(x) + \tan\left(\sqrt{x}\right)\right)$

b) $f'(x) = \frac{-\cot^2(x) - 1 - \frac{1}{\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{x^2}}{\sqrt{1-\left(\cot(x) + \frac{\sqrt{1-x^2}}{x}\right)^2}}$

c) $f'(x) = \frac{x^2\left(-\cot^2\left(\frac{x}{\sqrt{1-x^2}} + \arccos(x)\right) - 1\right)}{(1-x^2)^{\frac{3}{2}}}$

d) $f'(x) = -\left(-\frac{1}{x^2+1} - \frac{1}{\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{x^2}\right)\sin\left(\operatorname{arccot}(x) + \frac{\sqrt{1-x^2}}{x}\right)$

e) $f'(x) = \left(\tan^2(x) + 1 + \frac{e^{\sqrt{x}}}{2\sqrt{x}}\right)\cos\left(e^{\sqrt{x}} + \tan(x)\right)$

f) $f'(x) = -\frac{-\frac{1}{\sqrt{1-x^2}} + \frac{\cos(\ln(x))}{x}}{\sqrt{1-(\sin(\ln(x)) + \arccos(x))^2}}$

g) $f'(x) = -\left(-\frac{\sqrt{x}}{\sqrt{1-x^2}\left(\arcsin^2(x)+1\right)} + \frac{\operatorname{arccot}(\arcsin(x))}{2\sqrt{x}}\right)\sin\left(\sqrt{x}\operatorname{arccot}(\arcsin(x))\right)$

h) $f'(x) = \frac{(-\cot^2(\sin(x)) - 1)\cos(x)\arctan(x) + \frac{\cot(\sin(x))}{x^2+1}}{\sqrt{-\cot^2(\sin(x))\arctan^2(x) + 1}}$

i) $f'(x) = \left(\frac{\sin(\ln(x))}{x^2+1} + \frac{\cos(\ln(x))\arctan(x)}{x}\right)\left(\tan^2(\sin(\ln(x))\arctan(x)) + 1\right)$

j) $f'(x) = \frac{\frac{\arctan(\arcsin(x))}{\sqrt{1-x^2}} + \frac{\arcsin(x)}{\sqrt{1-x^2}\left(\arcsin^2(x)+1\right)}}{\arcsin^2(x)\arctan^2(\arcsin(x)) + 1}$