复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = \csc(\tan(x))$

b) $f(x) = e^{e^x}$

c) $f(x) = \sec\left(\sqrt{x}\right)$

d) $f(x) = \sin(\csc(x))$

e) $f(x) = \sec\left(\sqrt{x}\right)$

f) $f(x) = \sqrt{\csc(x)}$

g) $f(x) = \arcsin(e^x)$

h) $f(x) = \arccos(\operatorname{arccsc}(x))$

i) $f(x) = \arcsin^2(x)$

j) $f(x) = \ln(\operatorname{arccsc}(x))$

k) $f(x) = \arctan^2(x)$

l) $f(x) = \arctan(\arccos(x))$

m) $f(x) = \csc(\cot(x))$

n) $f(x) = \sec\left(x^3\right)$

o) $f(x) = \ln(\arctan(x))$

p) $f(x) = \arccos(\arcsin(x))$

q) $f(x) = \operatorname{arcsec}^3(x)$

r) $f(x) = \tan(\ln(x))$

2. 求下列函数的导数:

a) $f(x) = \cos\left(e^{\csc(x)}\right)$

b) $f(x) = \cos(\sec(\sin(x)))$

c) $f(x) = \cos(\cos(\sin(x)))$

d) $f(x) = e^{\tan\left(\sqrt{x}\right)}$

e) $f(x) = \arcsin\left(\operatorname{arccot}^2(x)\right)$

f) $f(x) = \arctan(\ln(\arctan(x)))$

g) $f(x) = \operatorname{arcsec}(\operatorname{arccot}(\operatorname{arccsc}(x)))$

h) $f(x) = \operatorname{arcsec}\left(\arccos^2(x)\right)$

i) $f(x) = \operatorname{arccot}(\operatorname{arccsc}(\cos(x)))$

j) $f(x) = \dfrac{1}{\sqrt{1 - \frac{1}{\arctan^2(x)}}\arctan(x)}$

k) $f(x) = \operatorname{arccot}\left(\dfrac{1}{x}\right)$

l) $f(x) = \ln\left(x^3\right)^3$

3. 求下列函数的导数：

a) $f(x) = \cot(e^x + \ln(x))$

b) $f(x) = \operatorname{arccot}(\arccos(x) + \arcsin(x))$

c) $f(x) = \cos(\tan(x) + \cot(x))$

d) $f(x) = \sin(\sin(x) + \arctan(x))$

e) $f(x) = \sqrt{\cos(x) + \tan(x)}$

f) $f(x) = \sqrt{2}\sqrt{\sin(x)}$

g) $f(x) = e^{\cos(x)\tan(x)}$

h) $f(x) = \arccos(\sin(x)\arccos(x))$

i) $f(x) = \arctan(\ln(x)\arccos(x))$

j) $f(x) = \ln(\cot(x)\,\mathrm{arccot}(x))$

k) $f(x) = \ln\left(\frac{\sqrt{x}}{\mathrm{arccot}(x)}\right)$

l) $f(x) = \tan\left(\frac{\arcsin(x)}{\sqrt{x}}\right)$

m) $f(x) = \arcsin\left(\frac{\sin(x)}{\sqrt{x}}\right)$

n) $f(x) = \arctan\left(\frac{\sqrt{x}}{\tan(x)}\right)$

4. 求下列函数的导数：

a) $f(x) = \ln\left(\sqrt{1-x^2} + \arcsin(x)\right)$

b) $f(x) = \sin(\ln(e^x) + \sin(x))$

c) $f(x) = \operatorname{arccot}(\tan(x) + \arcsin(\ln(x)))$

d) $f(x) = \cos\left(\sqrt{\cos(x)} + \tan(x)\right)$

e) $f(x) = \ln\left(e^{\operatorname{arccot}(x)} + \operatorname{arccot}(x)\right)$

f) $f(x) = \arctan(\cot(x) + \arctan(\sin(x)))$

g) $f(x) = \tan\left(\sqrt{x}\ln(\arccos(x))\right)$

h) $f(x) = \tan\left(\sqrt{1-x^2}e^x\right)$

i) $f(x) = \cos\left(\sqrt{x}\sin(\cos(x))\right)$

j) $f(x) = \cos\left(\sqrt[4]{x}\cos(x)\right)$

答案

1.

a) $f'(x) = -\left(\tan^2(x) + 1\right)\cot(\tan(x))\csc(\tan(x))$

b) $f'(x) = e^x e^{e^x}$

c) $f'(x) = \dfrac{\tan(\sqrt{x})\sec(\sqrt{x})}{2\sqrt{x}}$

d) $f'(x) = -\cos(\csc(x))\cot(x)\csc(x)$

e) $f'(x) = \dfrac{\tan(\sqrt{x})\sec(\sqrt{x})}{2\sqrt{x}}$

f) $f'(x) = -\dfrac{\cot(x)\sqrt{\csc(x)}}{2}$

g) $f'(x) = \dfrac{e^x}{\sqrt{1 - e^{2x}}}$

h) $f'(x) = \dfrac{1}{x^2\sqrt{1 - \frac{1}{x^2}}\sqrt{1 - \operatorname{arccsc}^2(x)}}$

i) $f'(x) = \dfrac{2\arcsin(x)}{\sqrt{1 - x^2}}$

j) $f'(x) = -\dfrac{1}{x^2\sqrt{1 - \frac{1}{x^2}}\operatorname{arccsc}(x)}$

k) $f'(x) = \dfrac{2\arctan(x)}{x^2 + 1}$

l) $f'(x) = -\dfrac{1}{\sqrt{1 - x^2}\left(\arccos^2(x) + 1\right)}$

m) $f'(x) = -\left(-\cot^2(x) - 1\right)\cot(\cot(x))\csc(\cot(x))$

n) $f'(x) = 3x^2\tan\left(x^3\right)\sec\left(x^3\right)$

o) $f'(x) = \dfrac{1}{(x^2 + 1)\arctan(x)}$

p) $f'(x) = -\dfrac{1}{\sqrt{1 - x^2}\sqrt{1 - \arcsin^2(x)}}$

q) $f'(x) = \dfrac{3\operatorname{arcsec}^2(x)}{x^2\sqrt{1 - \frac{1}{x^2}}}$

r) $f'(x) = \dfrac{\tan^2(\ln(x)) + 1}{x}$

2.

a) $f'(x) = e^{\csc(x)}\sin\left(e^{\csc(x)}\right)\cot(x)\csc(x)$

b) $f'(x) = -\sin(\sec(\sin(x)))\cos(x)\tan(\sin(x))\sec(\sin(x))$

c) $f'(x) = \sin(\sin(x))\sin(\cos(\sin(x)))\cos(x)$

d) $f'(x) = \dfrac{\left(\tan^2(\sqrt{x}) + 1\right)e^{\tan\left(\sqrt{x}\right)}}{2\sqrt{x}}$

e) $f'(x) = -\dfrac{2\operatorname{arccot}(x)}{\sqrt{1 - \operatorname{arccot}^4(x)}\,(x^2 + 1)}$

f) $f'(x) = \dfrac{1}{(x^2 + 1)\left(\ln(\arctan(x))^2 + 1\right)\arctan(x)}$

g) $f'(x) = \dfrac{1}{x^2\sqrt{1 - \frac{1}{x^2}}\sqrt{1 - \frac{1}{\operatorname{arccot}^2(\operatorname{arccsc}(x))}}\left(\operatorname{arccsc}^2(x) + 1\right)\operatorname{arccot}^2(\operatorname{arccsc}(x))}$

h) $f'(x) = -\frac{2}{\sqrt{1-x^2}\sqrt{1-\frac{1}{\arccos^4(x)}}\arccos^3(x)}$

i) $f'(x) = -\frac{\sin(x)}{\sqrt{1-\frac{1}{\cos^2(x)}}\left(\operatorname{arccsc}^2(\cos(x))+1\right)\cos^2(x)}$

j) $f'(x) = -\frac{1}{\sqrt{1-\frac{1}{\arctan^2(x)}}(x^2+1)\arctan^2(x)} - \frac{1}{\left(1-\frac{1}{\arctan^2(x)}\right)^{\frac{3}{2}}(x^2+1)\arctan^4(x)}$

k) $f'(x) = \frac{1}{x^2\left(1+\frac{1}{x^2}\right)}$

l) $f'(x) = \frac{9\ln(x^3)^2}{x}$

3.

a) $f'(x) = \left(e^x + \frac{1}{x}\right)\left(-\cot^2(e^x+\ln(x))-1\right)$

b) $f'(x) = 0$

c) $f'(x) = -\left(\tan^2(x)-\cot^2(x)\right)\sin(\tan(x)+\cot(x))$

d) $f'(x) = \left(\cos(x)+\frac{1}{x^2+1}\right)\cos(\sin(x)+\arctan(x))$

e) $f'(x) = \frac{-\frac{\sin(x)}{2}+\frac{\tan^2(x)}{2}+\frac{1}{2}}{\sqrt{\cos(x)+\tan(x)}}$

f) $f'(x) = \frac{\sqrt{2}\cos(x)}{2\sqrt{\sin(x)}}$

g) $f'(x) = \left(\left(\tan^2(x)+1\right)\cos(x)-\sin(x)\tan(x)\right)e^{\cos(x)\tan(x)}$

h) $f'(x) = -\frac{\cos(x)\arccos(x)-\frac{\sin(x)}{\sqrt{1-x^2}}}{\sqrt{-\sin^2(x)\arccos^2(x)+1}}$

i) $f'(x) = \frac{-\frac{\ln(x)}{\sqrt{1-x^2}}+\frac{\arccos(x)}{x}}{\ln(x)^2\arccos^2(x)+1}$

j) $f'(x) = \frac{(-\cot^2(x)-1)\operatorname{arccot}(x)-\frac{\cot(x)}{x^2+1}}{\cot(x)\operatorname{arccot}(x)}$

k) $f'(x) = \frac{\left(\frac{\sqrt{x}}{(x^2+1)\operatorname{arccot}^2(x)}+\frac{1}{2\sqrt{x}\operatorname{arccot}(x)}\right)\operatorname{arccot}(x)}{\sqrt{x}}$

l) $f'(x) = \left(\frac{1}{\sqrt{x}\sqrt{1-x^2}} - \frac{\arcsin(x)}{2x^{\frac{3}{2}}}\right)\left(\tan^2\left(\frac{\arcsin(x)}{\sqrt{x}}\right)+1\right)$

m) $f'(x) = \frac{\frac{\cos(x)}{\sqrt{x}} - \frac{\sin(x)}{2x^{\frac{3}{2}}}}{\sqrt{1-\frac{\sin^2(x)}{x}}}$

n) $f'(x) = \frac{\frac{\sqrt{x}\left(-\tan^2(x)-1\right)}{\tan^2(x)} + \frac{1}{2\sqrt{x}\tan(x)}}{\frac{x}{\tan^2(x)}+1}$

4.

a) $f'(x) = \frac{-\frac{x}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}}{\sqrt{1-x^2}+\arcsin(x)}$

b) $f'(x) = (\cos(x)+1)\cos(\ln(e^x)+\sin(x))$

c) $f'(x) = -\frac{\tan^2(x)+1+\frac{1}{x\sqrt{1-\ln(x)^2}}}{(\tan(x)+\arcsin(\ln(x)))^2+1}$

d) $f'(x) = -\left(-\frac{\sin(x)}{2\sqrt{\cos(x)}}+\tan^2(x)+1\right)\sin\left(\sqrt{\cos(x)}+\tan(x)\right)$

e) $f'(x) = \frac{-\frac{e^{\operatorname{arccot}(x)}}{x^2+1} - \frac{1}{x^2+1}}{e^{\operatorname{arccot}(x)}+\operatorname{arccot}(x)}$

f) $f'(x) = \frac{-\cot^2(x)-1+\frac{\cos(x)}{\sin^2(x)+1}}{(\cot(x)+\arctan(\sin(x)))^2+1}$

g) $f'(x) = \left(-\frac{\sqrt{x}}{\sqrt{1-x^2}\arccos(x)} + \frac{\ln(\arccos(x))}{2\sqrt{x}}\right)\left(\tan^2\left(\sqrt{x}\ln(\arccos(x))\right)+1\right)$

h) $f'(x) = \left(-\frac{xe^x}{\sqrt{1-x^2}} + \sqrt{1-x^2}e^x\right)\left(\tan^2\left(\sqrt{1-x^2}e^x\right)+1\right)$

i) $f'(x) = -\left(-\sqrt{x}\sin(x)\cos(\cos(x)) + \frac{\sin(\cos(x))}{2\sqrt{x}}\right)\sin\left(\sqrt{x}\sin(\cos(x))\right)$

j) $f'(x) = -\left(-\sqrt[4]{x}\sin(x) + \frac{\cos(x)}{4x^{\frac{3}{4}}}\right)\sin\left(\sqrt[4]{x}\cos(x)\right)$