复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x) = \cot(\ln(x))$

b) $f(x) = \sin(\csc(x))$

c) $f(x) = \ln(\ln(x))$

d) $f(x) = \sec(\cot(x))$

e) $f(x) = \sec(\tan(x))$

f) $f(x) = \csc(\ln(x))$

g) $f(x) = \arctan(\arcsin(x))$

h) $f(x) = \operatorname{arcsec}(\arctan(x))$

i) $f(x) = \operatorname{arccsc}(\operatorname{arcsec}(x))$

j) $f(x) = \arctan^2(x)$

k) $f(x) = \arcsin(\operatorname{arccsc}(x))$

l) $f(x) = \operatorname{arccsc}\left(x^2\right)$

m) $f(x) = e^{\sec(x)}$

n) $f(x) = \dfrac{1}{\sqrt{1-\frac{1}{x^2}}}$

o) $f(x) = \sin(\cos(x))$

p) $f(x) = \operatorname{arccot}\left(x^3\right)$

q) $f(x) = \arctan(\csc(x))$

r) $f(x) = \operatorname{arcsec}(\cot(x))$

2. 求下列函数的导数：

a) $f(x) = \ln(\cos(\cot(x)))$

b) $f(x) = \sec\left(\ln\left(\sqrt{x}\right)\right)$

c) $f(x) = \cos(\cot(\cot(x)))$

d) $f(x) = \csc(\ln(\sec(x)))$

e) $f(x) = \arctan(\operatorname{arccot}(\arccos(x)))$

f) $f(x) = \arctan(\arctan(\operatorname{arccsc}(x)))$

g) $f(x) = e^{\operatorname{arccot}(\operatorname{arccsc}(x))}$

h) $f(x) = \arccos(\arccos(\operatorname{arccot}(x)))$

i) $f(x) = e^{\sqrt{1+\frac{1}{x^2}}}$

j) $f(x) = \operatorname{arccot}(\arctan(\cot(x)))$

k) $f(x) = \arctan(\arctan(\operatorname{arccsc}(x)))$

l) $f(x) = \arccos(\operatorname{arccsc}(\tan(x)))$

3. 求下列函数的导数：

a) $f(x) = \operatorname{arccot}(\tan(x) + \cot(x))$

b) $f(x) = e^{e^x + \tan(x)}$

c) $f(x) = \arccos(\sin(x) + \operatorname{arccot}(x))$

d) $f(x) = \cot(2\cos(x))$

e) $f(x) = \ln\left(\sqrt{x} + \cos(x)\right)$

f) $f(x) = \cot(\ln(x) + \arctan(x))$

g) $f(x) = \arccos(\sin(x)\cos(x))$

h) $f(x) = \operatorname{arccot}(\cos(x)\operatorname{arccot}(x))$

i) $f(x) = \cot(\cos(x)\arccos(x))$

j) $f(x) = \cos(\ln(x)\sin(x))$

k) $f(x) = \cos\left(\frac{e^x}{\sin(x)}\right)$

l) $f(x) = \operatorname{arccot}\left(\frac{\operatorname{arccot}(x)}{\sin(x)}\right)$

m) $f(x) = \sqrt{\frac{\ln(x)}{\tan(x)}}$

n) $f(x) = \arccos\left(\frac{\operatorname{arccot}(x)}{\tan(x)}\right)$

4. 求下列函数的导数：

a) $f(x) = \sin(\arccos(x) + \arcsin(e^x))$

b) $f(x) = \operatorname{arccot}(e^x + \operatorname{arccot}(\arccos(x)))$

c) $f(x) = \arcsin\left(\sqrt{x} + x\right)$

d) $f(x) = \sqrt{\sin(x) + \arccos(\arctan(x))}$

e) $f(x) = \tan(\sin(x) + \arccos(\tan(x)))$

f) $f(x) = \cos(\arccos(\operatorname{arccot}(x)) + \operatorname{arccot}(x))$

g) $f(x) = \ln\left(e^x \sqrt{\cos(x)}\right)$

h) $f(x) = \arccos\left(\sqrt{1 - x^2}\ln(x)\right)$

i) $f(x) = \tan(\arcsin(x)\arcsin(\operatorname{arccot}(x)))$

j) $f(x) = \sin\left(\frac{\sqrt{1 - x^2}\arcsin(x)}{x}\right)$

答案

1.

a) $f'(x) = \frac{-\cot^2(\ln(x)) - 1}{x}$

b) $f'(x) = -\cos(\csc(x))\cot(x)\csc(x)$

c) $f'(x) = \frac{1}{x\ln(x)}$

d) $f'(x) = \left(-\cot^2(x) - 1\right)\tan(\cot(x))\sec(\cot(x))$

e) $f'(x) = \left(\tan^2(x) + 1\right)\tan(\tan(x))\sec(\tan(x))$

f) $f'(x) = -\frac{\cot(\ln(x))\csc(\ln(x))}{x}$

g) $f'(x) = \frac{1}{\sqrt{1-x^2}\left(\arcsin^2(x) + 1\right)}$

h) $f'(x) = \frac{1}{\sqrt{1 - \frac{1}{\arctan^2(x)}}(x^2+1)\arctan^2(x)}$

i) $f'(x) = -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1 - \frac{1}{\operatorname{arcsec}^2(x)}}\operatorname{arcsec}^2(x)}$

j) $f'(x) = \frac{2\arctan(x)}{x^2+1}$

k) $f'(x) = -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arccsc}^2(x)}}$

l) $f'(x) = -\frac{2}{x^3\sqrt{1-\frac{1}{x^4}}}$

m) $f'(x) = e^{\sec(x)}\tan(x)\sec(x)$

n) $f'(x) = -\frac{1}{x^3\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}}$

o) $f'(x) = -\sin(x)\cos(\cos(x))$

p) $f'(x) = -\frac{3x^2}{x^6+1}$

q) $f'(x) = -\frac{\cot(x)\csc(x)}{\csc^2(x)+1}$

r) $f'(x) = \frac{-\cot^2(x) - 1}{\sqrt{1-\frac{1}{\cot^2(x)}}\cot^2(x)}$

2.

a) $f'(x) = -\frac{(-\cot^2(x) - 1)\sin(\cot(x))}{\cos(\cot(x))}$

b) $f'(x) = \frac{\tan(\ln(\sqrt{x}))\sec(\ln(\sqrt{x}))}{2x}$

c) $f'(x) = -\left(-\cot^2(x) - 1\right)\left(-\cot^2(\cot(x)) - 1\right)\sin(\cot(\cot(x)))$

d) $f'(x) = -\tan(x)\cot(\ln(\sec(x)))\csc(\ln(\sec(x)))$

e) $f'(x) = \frac{1}{\sqrt{1-x^2}(\arccos^2(x)+1)(\operatorname{arccot}^2(\arccos(x))+1)}$

f) $f'(x) = -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}(\operatorname{arccsc}^2(x)+1)(\arctan^2(\operatorname{arccsc}(x))+1)}$

g) $f'(x) = \frac{e^{\operatorname{arccot}(\operatorname{arccsc}(x))}}{x^2\sqrt{1-\frac{1}{x^2}}(\operatorname{arccsc}^2(x)+1)}$

h) $f'(x) = -\frac{1}{\sqrt{1-\operatorname{arccos}^2(\operatorname{arccot}(x))}\sqrt{1-\operatorname{arccot}^2(x)}(x^2+1)}$

i) $f'(x) = -\frac{e^{\sqrt{1+\frac{1}{x^2}}}}{x^3\sqrt{1+\frac{1}{x^2}}}$

j) $f'(x) = -\frac{-\cot^2(x)-1}{(\cot^2(x)+1)(\arctan^2(\cot(x))+1)}$

k) $f'(x) = -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}(\operatorname{arccsc}^2(x)+1)(\arctan^2(\operatorname{arccsc}(x))+1)}$

l) $f'(x) = \frac{\tan^2(x)+1}{\sqrt{1-\frac{1}{\tan^2(x)}}\sqrt{1-\operatorname{arccsc}^2(\tan(x))}\tan^2(x)}$

3.

a) $f'(x) = -\frac{\tan^2(x)-\cot^2(x)}{(\tan(x)+\cot(x))^2+1}$

b) $f'(x) = \left(e^x+\tan^2(x)+1\right)e^{e^x+\tan(x)}$

c) $f'(x) = -\frac{\cos(x)-\frac{1}{x^2+1}}{\sqrt{1-(\sin(x)+\operatorname{arccot}(x))^2}}$

d) $f'(x) = -2\left(-\cot^2(2\cos(x))-1\right)\sin(x)$

e) $f'(x) = \frac{-\sin(x)+\frac{1}{2\sqrt{x}}}{\sqrt{x}+\cos(x)}$

f) $f'(x) = \left(\frac{1}{x^2+1}+\frac{1}{x}\right)\left(-\cot^2(\ln(x)+\arctan(x))-1\right)$

g) $f'(x) = -\frac{-\sin^2(x)+\cos^2(x)}{\sqrt{-\sin^2(x)\cos^2(x)+1}}$

h) $f'(x) = -\frac{-\sin(x)\operatorname{arccot}(x)-\frac{\cos(x)}{x^2+1}}{\cos^2(x)\operatorname{arccot}^2(x)+1}$

i) $f'(x) = \left(-\sin(x)\arccos(x)-\frac{\cos(x)}{\sqrt{1-x^2}}\right)\left(-\cot^2(\cos(x)\arccos(x))-1\right)$

j) $f'(x) = -\left(\ln(x)\cos(x)+\frac{\sin(x)}{x}\right)\sin(\ln(x)\sin(x))$

k) $f'(x) = -\left(\frac{e^x}{\sin(x)}-\frac{e^x\cos(x)}{\sin^2(x)}\right)\sin\left(\frac{e^x}{\sin(x)}\right)$

l) $f'(x) = -\frac{-\frac{\cos(x)\operatorname{arccot}(x)}{\sin^2(x)} - \frac{1}{(x^2+1)\sin(x)}}{1 + \frac{\operatorname{arccot}^2(x)}{\sin^2(x)}}$

m) $f'(x) = \frac{\sqrt{\frac{\ln(x)}{\tan(x)}}\left(\frac{\left(-\tan^2(x)-1\right)\ln(x)}{2\tan^2(x)} + \frac{1}{2x\tan(x)}\right)\tan(x)}{\ln(x)}$

n) $f'(x) = -\frac{\frac{\left(-\tan^2(x)-1\right)\operatorname{arccot}(x)}{\tan^2(x)} - \frac{1}{(x^2+1)\tan(x)}}{\sqrt{1 - \frac{\operatorname{arccot}^2(x)}{\tan^2(x)}}}$

4.

a) $f'(x) = \left(\frac{e^x}{\sqrt{1-e^{2x}}} - \frac{1}{\sqrt{1-x^2}}\right)\cos\left(\arccos(x) + \arcsin(e^x)\right)$

b) $f'(x) = -\frac{e^x + \frac{1}{\sqrt{1-x^2}(\arccos^2(x)+1)}}{\left(e^x + \operatorname{arccot}(\arccos(x))\right)^2 + 1}$

c) $f'(x) = \frac{1 + \frac{1}{2\sqrt{x}}}{\sqrt{1 - \left(\sqrt{x} + x\right)^2}}$

d) $f'(x) = \frac{\frac{\cos(x)}{2} - \frac{1}{2\sqrt{1-\arctan^2(x)}(x^2+1)}}{\sqrt{\sin(x) + \arccos(\arctan(x))}}$

e) $f'(x) = \left(\cos(x) - \frac{\tan^2(x)+1}{\sqrt{1-\tan^2(x)}}\right)\left(\tan^2\left(\sin(x) + \arccos(\tan(x))\right) + 1\right)$

f) $f'(x) = -\left(-\frac{1}{x^2+1} + \frac{1}{\sqrt{1-\operatorname{arccot}^2(x)}\,(x^2+1)}\right)\sin\left(\arccos(\operatorname{arccot}(x)) + \operatorname{arccot}(x)\right)$

g) $f'(x) = \frac{\left(-\frac{e^x\sin(x)}{2\sqrt{\cos(x)}} + e^x\sqrt{\cos(x)}\right)e^{-x}}{\sqrt{\cos(x)}}$

h) $f'(x) = -\frac{-\frac{x\ln(x)}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{x}}{\sqrt{-(1-x^2)\ln(x)^2 + 1}}$

i) $f'(x) = \left(-\frac{\arcsin(x)}{\sqrt{1-\operatorname{arccot}^2(x)}\,(x^2+1)} + \frac{\arcsin(\operatorname{arccot}(x))}{\sqrt{1-x^2}}\right)\left(\tan^2\left(\arcsin(x)\arcsin(\operatorname{arccot}(x))\right) + 1\right)$

j) $f'(x) = \left(-\frac{\arcsin(x)}{\sqrt{1-x^2}} + \frac{1}{x} - \frac{\sqrt{1-x^2}\arcsin(x)}{x^2}\right)\cos\left(\frac{\sqrt{1-x^2}\arcsin(x)}{x}\right)$