复合函数的求导法则

练习

1. 求下列函数的导数：

a) $f(x)=\sqrt{\csc(x)}$

b) $f(x)=e^{e^{x}}$

c) $f(x)=\sqrt{\sec(x)}$

d) $f(x)=\cos(\cot(x))$

e) $f(x)=e^{\sec(x)}$

f) $f(x)=\tan\left(\sqrt{x}\right)$

g) $f(x)=\operatorname{arccot}\left(x^2\right)$

h) $f(x)=\operatorname{arcsec}(\arctan(x))$

i) $f(x)=\operatorname{arccot}(\operatorname{arccsc}(x))$

j) $f(x)=\operatorname{arccsc}\left(x^2\right)$

k) $f(x) = \arcsin\left(\text{arccot}\left(x\right)\right)$

l) $f(x) = \text{arcsec}\left(\ln\left(x\right)\right)$

m) $f(x) = x\sqrt{1 - \dfrac{1}{x^2}}$

n) $f(x) = \sqrt{1 - \dfrac{1}{x^2}}$

o) $f(x) = \arcsin^3\left(x\right)$

p) $f(x) = \csc\left(\cot\left(x\right)\right)$

q) $f(x) = \dfrac{1}{\sqrt{1 + \frac{1}{x^2}}}$

r) $f(x) = \cos\left(e^x\right)$

2. 求下列函数的导数:

a) $f(x) = \sec\left(\cot\left(\ln\left(x\right)\right)\right)$

b) $f(x) = \csc\left(\sqrt{\sec\left(x\right)}\right)$

c) $f(x) = \sin\left(\tan\left(\csc\left(x\right)\right)\right)$

d) $f(x) = \csc\left(\tan\left(\sec\left(x\right)\right)\right)$

e) $f(x) = e^{\operatorname{arccot}(\operatorname{arcsec}(x))}$

f) $f(x) = e^{\operatorname{arccot}(\operatorname{arcsec}(x))}$

g) $f(x) = \arccos\left(e^{\arcsin(x)}\right)$

h) $f(x) = \arccos\left(\operatorname{arccsc}^2(x)\right)$

i) $f(x) = \sin(\sec(\sec(x)))$

j) $f(x) = \sin(\sin(\sec(x)))$

k) $f(x) = \dfrac{1}{\operatorname{arccot}(x)}$

l) $f(x) = \cos^3(\cot(x))$

3. 求下列函数的导数：

a) $f(x) = \cot\left(\sqrt{x} + \cos(x)\right)$

b) $f(x) = \ln(\sin(x) + \arccos(x))$

c) $f(x) = \sqrt{e^x + \operatorname{arccot}(x)}$

d) $f(x) = \sqrt{e^x + \arccos(x)}$

e) $f(x) = \ln(\sin(x) + \cot(x))$

f) $f(x) = \cos(\tan(x) + \arcsin(x))$

g) $f(x) = \arcsin(\operatorname{arccot}(x)\arctan(x))$

h) $f(x) = \arccos(\tan(x)\cot(x))$

i) $f(x) = \ln\left(\sqrt{x}e^{x}\right)$

j) $f(x) = \cos\left(\sqrt{x}\arccos(x)\right)$

k) $f(x) = \operatorname{arccot}\left(\frac{\cos(x)}{\sqrt{x}}\right)$

l) $f(x) = \cos\left(\frac{\ln(x)}{\tan(x)}\right)$

m) $f(x) = \cot\left(\frac{\operatorname{arccot}(x)}{\tan(x)}\right)$

n) $f(x) = \cot\left(e^{-x}\sin(x)\right)$

4. 求下列函数的导数：

a) $f(x) = \cos\left(\ln\left(\sqrt{x}\right) + \arccos(x)\right)$

b) $f(x) = \arcsin(\ln(\sin(x)) + \arctan(x))$

c) $f(x) = \operatorname{arccot}(\ln(\arctan(x)) + \arcsin(x))$

d) $f(x) = e^{e^x + e^{\sin(x)}}$

e) $f(x) = \arccos\left(\sqrt{x} + \cot(\tan(x))\right)$

f) $f(x) = \cot(\cos(e^x) + \arctan(x))$

g) $f(x) = \arctan(\arccos(\sin(x))\arcsin(x))$

h) $f(x) = \ln(\cos(x)\operatorname{arccot}(\arccos(x)))$

i) $f(x) = \operatorname{arccot}\left(\sqrt{\ln(x)}\operatorname{arccot}(x)\right)$

j) $f(x) = \arcsin(e^x \arcsin(\cot(x)))$

答案

1.

a) $f'(x) = -\frac{\cot(x)\sqrt{\csc(x)}}{2}$

b) $f'(x) = e^x e^{e^x}$

c) $f'(x) = \frac{\tan(x)\sqrt{\sec(x)}}{2}$

d) $f'(x) = -\left(-\cot^2(x) - 1\right)\sin(\cot(x))$

e) $f'(x) = e^{\sec(x)}\tan(x)\sec(x)$

f) $f'(x) = \frac{\tan^2(\sqrt{x}) + 1}{2\sqrt{x}}$

g) $f'(x) = -\frac{2x}{x^4+1}$

h) $f'(x) = \frac{1}{\sqrt{1 - \frac{1}{\arctan^2(x)}}\,(x^2+1)\arctan^2(x)}$

i) $f'(x) = \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}\,(\operatorname{arccsc}^2(x)+1)}$

j) $f'(x) = -\frac{2}{x^3\sqrt{1-\frac{1}{x^4}}}$

k) $f'(x) = -\frac{1}{\sqrt{1-\operatorname{arccot}^2(x)}\,(x^2+1)}$

l) $f'(x) = \frac{1}{x\sqrt{1-\frac{1}{\ln(x)^2}}\ln(x)^2}$

m) $f'(x) = \sqrt{1-\frac{1}{x^2}} + \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}$

n) $f'(x) = \frac{1}{x^3\sqrt{1-\frac{1}{x^2}}}$

o) $f'(x) = \frac{3\arcsin^2(x)}{\sqrt{1-x^2}}$

p) $f'(x) = -\left(-\cot^2(x) - 1\right)\cot(\cot(x))\csc(\cot(x)$

q) $f'(x) = \frac{1}{x^3\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$

r) $f'(x) = -e^x\sin(e^x)$

2.

a) $f'(x) = \frac{(-\cot^2(\ln(x)) - 1)\tan(\cot(\ln(x)))\sec(\cot(\ln(x)))}{x}$

b) $f'(x) = -\frac{\tan(x)\cot\left(\sqrt{\sec(x)}\right)\csc\left(\sqrt{\sec(x)}\right)\sqrt{\sec(x)}}{2}$

c) $f'(x) = -\left(\tan^2(\csc(x)) + 1\right)\cos(\tan(\csc(x)))\cot(x)\csc(x)$

d) $f'(x) = -\left(\tan^2(\sec(x)) + 1\right)\tan(x)\cot(\tan(\sec(x)))\csc(\tan(\sec(x)))\sec(x)$

e) $f'(x) = -\frac{e^{\operatorname{arccot}(\operatorname{arcsec}(x))}}{x^2\sqrt{1-\frac{1}{x^2}}\,(\operatorname{arcsec}^2(x)+1)}$

f) $f'(x) = -\frac{e^{\operatorname{arccot}(\operatorname{arcsec}(x))}}{x^2\sqrt{1-\frac{1}{x^2}}\,(\operatorname{arcsec}^2(x)+1)}$

g) $f'(x) = -\frac{e^{\arcsin(x)}}{\sqrt{1-x^2}\sqrt{1-e^{2\arcsin(x)}}}$

h) $f'(x) = \frac{2\operatorname{arccsc}(x)}{x^2\sqrt{1-\frac{1}{x^2}}\sqrt{1-\operatorname{arccsc}^4(x)}}$

i) $f'(x) = \cos(\sec(\sec(x)))\tan(x)\tan(\sec(x))\sec(x)\sec(\sec(x))$

j) $f'(x) = \cos(\sin(\sec(x)))\cos(\sec(x))\tan(x)\sec(x)$

k) $f'(x) = \frac{1}{(x^2+1)\operatorname{arccot}^2(x)}$

l) $f'(x) = -3\left(-\cot^2(x) - 1\right)\sin(\cot(x))\cos^2(\cot(x))$

3.

a) $f'(x) = \left(-\sin(x) + \frac{1}{2\sqrt{x}}\right)\left(-\cot^2\left(\sqrt{x}+\cos(x)\right) - 1\right)$

b) $f'(x) = \frac{\cos(x) - \frac{1}{\sqrt{1-x^2}}}{\sin(x) + \arccos(x)}$

c) $f'(x) = \frac{\frac{e^x}{2} - \frac{1}{2(x^2+1)}}{\sqrt{e^x + \operatorname{arccot}(x)}}$

d) $f'(x) = \frac{\frac{e^x}{2} - \frac{1}{2\sqrt{1-x^2}}}{\sqrt{e^x + \arccos(x)}}$

e) $f'(x) = \frac{\cos(x) - \cot^2(x) - 1}{\sin(x) + \cot(x)}$

f) $f'(x) = -\left(\tan^2(x) + 1 + \frac{1}{\sqrt{1-x^2}}\right)\sin(\tan(x) + \arcsin(x))$

g) $f'(x) = \frac{\frac{\operatorname{arccot}(x)}{x^2+1} - \frac{\arctan(x)}{x^2+1}}{\sqrt{-\operatorname{arccot}^2(x)\arctan^2(x) + 1}}$

h) $f'(x) = -\frac{(\tan^2(x)+1)\cot(x) + (-\cot^2(x)-1)\tan(x)}{\sqrt{-\tan^2(x)\cot^2(x)+1}}$

i) $f'(x) = \frac{\left(\sqrt{x}e^x + \frac{e^x}{2\sqrt{x}}\right)e^{-x}}{\sqrt{x}}$

j) $f'(x) = -\left(-\frac{\sqrt{x}}{\sqrt{1-x^2}} + \frac{\arccos(x)}{2\sqrt{x}}\right)\sin\left(\sqrt{x}\arccos(x)\right)$

k) $f'(x) = -\frac{-\frac{\sin(x)}{\sqrt{x}} - \frac{\cos(x)}{2x^{\frac{3}{2}}}}{1 + \frac{\cos^2(x)}{x}}$

l) $f'(x) = -\left(\frac{\left(-\tan^2(x) - 1\right)\ln(x)}{\tan^2(x)} + \frac{1}{x\tan(x)}\right)\sin\left(\frac{\ln(x)}{\tan(x)}\right)$

m) $f'(x) = \left(\frac{\left(-\tan^2(x) - 1\right)\operatorname{arccot}(x)}{\tan^2(x)} - \frac{1}{(x^2+1)\tan(x)}\right)\left(-\cot^2\left(\frac{\operatorname{arccot}(x)}{\tan(x)}\right) - 1\right)$

n) $f'(x) = \left(-e^{-x}\sin(x) + e^{-x}\cos(x)\right)\left(-\cot^2\left(e^{-x}\sin(x)\right) - 1\right)$

4.

a) $f'(x) = -\left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{2x}\right)\sin\left(\ln\left(\sqrt{x}\right) + \arccos(x)\right)$

b) $f'(x) = \frac{\frac{\cos(x)}{\sin(x)} + \frac{1}{x^2+1}}{\sqrt{1 - \left(\ln(\sin(x)) + \arctan(x)\right)^2}}$

c) $f'(x) = -\frac{\frac{1}{(x^2+1)\arctan(x)} + \frac{1}{\sqrt{1-x^2}}}{\left(\ln(\arctan(x)) + \arcsin(x)\right)^2 + 1}$

d) $f'(x) = \left(e^x + e^{\sin(x)}\cos(x)\right)e^{e^x + e^{\sin(x)}}$

e) $f'(x) = -\frac{\left(\tan^2(x) + 1\right)\left(-\cot^2(\tan(x)) - 1\right) + \frac{1}{2\sqrt{x}}}{\sqrt{1 - \left(\sqrt{x} + \cot(\tan(x))\right)^2}}$

f) $f'(x) = \left(-e^x\sin(e^x) + \frac{1}{x^2+1}\right)\left(-\cot^2\left(\cos(e^x) + \arctan(x)\right) - 1\right)$

g) $f'(x) = \frac{-\frac{\cos(x)\arcsin(x)}{\sqrt{1-\sin^2(x)}} + \frac{\arccos(\sin(x))}{\sqrt{1-x^2}}}{\arccos^2(\sin(x))\arcsin^2(x) + 1}$

h) $f'(x) = \frac{-\sin(x)\operatorname{arccot}(\arccos(x)) + \frac{\cos(x)}{\sqrt{1-x^2}\left(\arccos^2(x)+1\right)}}{\cos(x)\operatorname{arccot}(\arccos(x))}$

i) $f'(x) = -\frac{-\frac{\sqrt{\ln(x)}}{x^2+1} + \frac{\operatorname{arccot}(x)}{2x\sqrt{\ln(x)}}}{\ln(x)\operatorname{arccot}^2(x) + 1}$

j) $f'(x) = \frac{e^x\arcsin(\cot(x)) + \frac{\left(-\cot^2(x) - 1\right)e^x}{\sqrt{1-\cot^2(x)}}}{\sqrt{-e^{2x}\arcsin^2(\cot(x)) + 1}}$